



# Hamilton Institute

Global analysis and synthesis of networks of oscillators: a  
dissipativity approach

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Wednesday, June 25th, 2008

## *Abstract*

The main theme of this presentation concerns the global (as opposed to local) analysis of stable limit cycle oscillations in dynamical systems. Dynamical systems that exhibit robust nonlinear oscillations are called oscillators. Oscillators are ubiquitous in physical, biological, biochemical, and electromechanical systems. Detailed models of oscillators abound in the literature, most frequently in the form of a set of nonlinear differential equations whose solutions robustly converge to a limit cycle oscillation. Local stability analysis of oscillators is possible by means of Floquet theory but global stability analysis is usually restricted to low-dimensional (second order) models. For these low dimensional models, global analysis is performed by using specific planar tools (phase plane methods, Poincaré-Bendixson theorem, etc.) which do not generalise easily to high-dimensional models. As a consequence, global limit cycle analysis of models of dimension higher than 2 is quite hard since there currently exists no general analysis method. This lack of general analysis methods typically forces complex models of oscillators to be studied through numerical simulations. Although numerical simulations of these models may give an insight into their behaviour, a more in-depth understanding is generally impeded by the complexity of the models and the challenge of their rigorous global stability analysis. Moreover, even in the case of low-dimensional models, the planar methods used for their (global) analysis do not generalise to the analysis of their interconnection into a network. These considerations show the need for developing dimension independent methods that allow the global analysis of oscillators, either isolated or in interconnection.

**Venue:** Seminar Room, Hamilton Institute, Science Building,  
NUI Maynooth

**Time:** 2.00 - 3.00pm (followed by tea/coffee)

Travel directions are available at [www.hamilton.ie](http://www.hamilton.ie)



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In particular, this presentation aims at showing that the dissipativity theory introduced by Willems can be extended to allow global stability analysis of limit cycles in Lure-type models of passive oscillators and networks of interconnected passive oscillators.

As will be shown, a global limit cycle in such systems either results from a supercritical Hopf bifurcation, or from a supercritical pitchfork bifurcation that yields a globally bistable system which is then easily turned into a relaxation oscillation. The first scenario provides a generalisation of the Van der Pol oscillators. Its energy interpretation fits the qualitative description of a lossless exchange of energy between two storage elements, regulated by a locally active but globally dissipative element. The second scenario provides a generalisation of FitzHugh-Nagumo oscillators. Its energy interpretation fits the qualitative description of many oscillation mechanisms in biology, viewed as periodic switches between two quasi-steady-states.

From the global analysis point of view, the main results of this research deal with the implications of this extended dissipativity theory for:

1. the global stability analysis of limit cycle oscillations in isolated passive oscillators;
2. the global stability analysis of limit cycle oscillations in networks of passive oscillators;
3. the global analysis of synchronised limit cycle oscillations in networks of identical passive oscillators.

From the synthesis point of view, the structure of passive oscillators suggests a method for the design of a nonlinear parametric proportional-integral controller aimed at the generation of limit cycle oscillations with large basins of attraction in stabilisable (through passivation) nonlinear systems.

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