Robust stabilization by linear output feedback

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(joint work with Mark French and Achim Ilchmann)

December 2008

We consider linear systems (A, b, c) of the form

$$\dot{x} = Ax + bu\,, \quad y = cx\,, \tag{1}$$

with $A \in \mathbb{R}^{n \times n}$, $b, c^T \in \mathbb{R}^n$, for which the system matrices may be unknown, but the system's relative degree $r \in \mathbb{N}$ and the sign of the high frequency gain $cA^{r-1}b$ are known, and moreover the system has exponentially stable zero dynamics. Under this conditions we show that, for any $k_1, \ldots, k_r \in \mathbb{R}$, which satisfy that $\sum_{i=0}^{r-1} k_{i+1} s^i$ is Hurwitz, and sufficiently large $\kappa > 0$, the output derivative feedback C

$$u(t) = -\kappa \sum_{i=0}^{r-1} \kappa^{r-i} k_{i+1} y^{(i)}(t)\,, \tag{2}$$

applied to (1) gives an exponentially stable closed-loop $[(A, b, c), C]$. If the output y but not its derivatives are available one has to approximate the derivatives with, for example, Euler's method: for small $h > 0$ let

$$y^{(i)}(t) \approx \Delta_h^i(y)(t)\,,$$

where $\Delta_h^i(y) = \begin{cases} \Delta_h^{i-1}(\Delta_h(y)) & i \geq 1 \\ y & i = 0 \end{cases}$ and $\Delta_h(y)(t) = \frac{1}{h}\big(y(t) - y(t-h)\big)$.

We show existence of a constant $h^* > 0$, such that for all $h \in (0, h^*)$ the feedback (2) can be replaced by the output delay feedback $C[h]$

$$u(t) = -\kappa \sum_{i=0}^{r-1} \kappa^{r-i} k_{i+1} \Delta_h^i(y)(t)\,, \tag{3}$$

such that the closed-loop $[(A, b, c), C[h]]$ remains stable. To proof this we apply the concept of the „gap metric“ (see Georgiou & Smith, IEEE Trans. AC 42(9) 1200–1221, 1997). We also show that stabilization is robust under L^p-input and $W^{r,p}$-output disturbances, where $W^{r,p}$ is the Sobolev space of L^p-functions, which first r derivatives are also in L^p.