GLOBAL GAIN-SCHEDULING CONTROL FOR VARIABLE SPEED WIND TURBINES

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ABSTRACT It is shown that the nonlinear rotor aerodynamics of typical medium and large-scale variable speed wind turbines exhibit a separability property similar to that previously established for constant speed wind turbines. The importance of this result is that it establishes that from a control viewpoint a variable speed wind turbine can be viewed as a nonlinear system with static nonlinearity, h, subject to the external wind disturbance. It is emphasised that the function h is *independent* of the wind speed. This immediately suggests that the aerodynamic nonlinearity of the rotor can be accommodated by incorporating the inverse function h^{-1} within the controller. With the plant dynamics globally linearised in this way, the rest of the controller can be designed using linear methods.
Keywords Control Systems, Gain-scheduling, Nonlinear Control, Variable Speed HAWTs

1.INTRODUCTION

The dynamics of pitch regulated *constant speed* wind turbines are strongly nonlinear since the rotor aerodynamic torque is nonlinearly dependent on the rotor blade pitch angle and the wind speed. To obtain acceptable performance, the pitch control system must cater for this nonlinearity. The usual approach is by gain-scheduling whereby the turbine dynamics are linearised about a series of equilibrium operating points. It can be shown that the linearised models are essentially the same at every operating point except for a gain which varies with the operating point (the gain is simply the partial derivative of aerodynamic torque with respect to pitch angle). In order to accommodate the varying plant gain, the reciprocal of the varying gain is included in the controller. Typically, the varying gain is scheduled with respect to pitch angle since a direct wind speed measurement is, of course, not possible. A nonlinear gain-scheduled controller designed in the above manner is really only valid provided the operating state of the wind turbine is slowly varying and, in addition, locally confined to the equilibrium operating points. However, neither of these two requirements is met by constant speed wind turbines. Nevertheless, provided an appropriate controller implementation is adopted, the gain-scheduled controller is observed to be globally valid; that is, to operate as intended in all circumstances (Leith & Leithead 1997). (It should be noted that the dynamic behaviour of the controller depends strongly on its realisation and the operation of other realisations is poorer). The reason for the unexpectedly good operation of the nonlinear gain-scheduled controllers has been investigated and identified. The controllers are indeed globally valid and systematic procedures for choosing the realisations, which operate globally as intended, have been determined (Leith & Leithead 1997).

The same issue must be addressed for pitch regulated *variable speed* wind turbines. In this context, the aerodynamic torque is strongly dependent not only on rotor blade pitch angle and wind speed but also on rotor speed. Consequently, the linear models, local to the equilibrium operating points, contain a varying low frequency pole in addition to a varying gain. Consequently, the design of the nonlinear gain- scheduled pitch controller is rather more complex than in the constant speed case. The purpose of this paper is to investigate the nonlinear aspects of the pitch controller design for variable speed wind turbines with the aim of determining, if possible, a globally valid realisation.

2. NONLINEAR AERODYNAMICS

Although there is no such thing as the 'wind speed' experienced by a wind turbine, since the rotor is subject to a spatially and temporally distributed wind field, it may be considered to experience an effective wind speed which, in some sense, is an average over the rotor disc. (It should be noted that this makes a direct measurement of wind speed impossible). In a variable speed wind turbine the aerodynamic torque, T, depends nonlinearly on the pitch angle, p, the rotor speed, Ω, and the effective wind speed, V, as in figure 1a; that is,

$$T = T(p,\Omega,V) \qquad (1)$$

(Throughout this paper it is assumed that T is differentiable as required). For each effective wind speed, V_o, above rated wind speed, at the rated rotor speed, Ω_o, the rated aerodynamic torque, T_o, is attained at a unique pitch angle, p_{V_o}. It follows from Taylor series expansion theory that locally to a specific equilibrium operating point, (p_{V_o},Ω_o,V_o), the nonlinearity (1) may be linearised as depicted in figure 1b, where δ indicates perturbations about the nominal values. The partial derivatives, at the equilibrium points, of the aerodynamic torque with respect to pitch angle, rotor speed and wind speed, $\partial T/\partial p$, $\partial T/\partial \Omega$ and $\partial T/\partial V$ respectively, are strongly dependent on the operating point indicating that the behaviour of the aerodynamic torque varies considerably over the operational envelope of the wind turbine. With regard to the linearised plant dynamics locally to an equilibrium operating point, the variation in $\partial T/\partial p$ induces a variation in the plant gain with operating point whilst the variation in $\partial T/\partial \Omega$ induces a variation in one of the low frequency poles of the plant. The variation in $\partial T/\partial V$ affects the magnitude of the torque disturbance, induced by changes in the effective wind speed, at different operating points.

3. CONVENTIONAL GAIN-SCHEDULING

The gain-scheduling approach to compensation of the nonlinear aerodynamics is to incorporate the reciprocal of the aerodynamic 'gain', $\partial T/\partial p(p_{V_o},\Omega_o,V_o)$, within the controller and to include a zero which depends on $\partial T/\partial \Omega(p_{V_o},\Omega_o,V_o)$. Frequently, the controller is simplified by using a fixed zero and employing the robustness provided by high-gain feedback to accommodate the variation in the plant dynamics induced by the variations in $\partial T/\partial \Omega$. In either case, the controller is scheduled with respect to a variable which parameterises the locus of equilibrium operating points. Since scheduling on a direct measurement of wind speed is impossible, the pitch angle may be employed provided the pitch angle is always sufficiently close to p_V. Note that there is no scheduling with respect to rotor speed since this is the same at every equilibrium operating point. When the system is sufficiently weakly nonlinear (or, equivalently, the operating point of the system remains sufficiently close to the locus of equilibrium operating points and varies sufficiently slowly), the stability of the gain-scheduled nonlinear system may be inferred from the stability of the members of the family of linear systems consisting of the linearisations of the nonlinear system at each equilibrium operating point (*e.g.* Khalil & Kokotovic 1991).

However, the wind speed fluctuations are highly stochastic and the operating point of the wind turbine varies rapidly and continuously over the whole operating envelope. Whilst the bandwidth of the linearised closed-loop system is typically about 1 r/s, the operating point might cover its full range, corresponding to an order of magnitude or greater change in the aerodynamic gain, in a few seconds. Moreover, large, rapid fluctuations in wind speed are common, in particular gusts; that is, steady increases or decreases in the wind speed which persist for relatively long periods and produce substantial and prolonged perturbations from equilibrium. Hence, *a priori*, the system cannot be assumed to be weakly nonlinear and the emphasis must be on the nonlinear and non-local behaviour and performance of the controller. A nonlinear analysis of the nonlinear gain-scheduled controller is required.

3. SEPARABILITY OF THE NONLINEAR AERODYNAMICS

It is known that for constant speed machines the dependence, on pitch angle and wind speed, of the aerodynamic torque over the operational envelope can be explicitly separated (Leith & Leithead 1997) as

$$T(p,V) = h(p)-g(V) \qquad (2)$$

for nonlinear functions h and g satisfying

$$\frac{dh}{dp} = \frac{\partial T}{\partial p}(p_{V_o}, V_o); \qquad \frac{dg}{dV} = -\frac{\partial T}{\partial V}(p_{V_o}, V_o)$$

This separability of the nonlinear aerodynamics is a quite general feature because there are underlying physical reasons why the representation (2) should hold for all wind turbines. The aerodynamic torque largely stems from the outer third of the rotor but, in this region, the velocity of the blade is much greater than the wind velocity. It follows that the direction of the wind velocity, relative to the blades, changes almost linearly as the wind speed varies but its magnitude changes little. Hence, the aerodynamic torque is largely a function of the angle of attack of the wind on the outer third of the blades, which is simply the difference in the direction of the relative velocity of the wind and the pitch angle. The importance of this result is that it establishes that from a control viewpoint the wind turbine can be viewed as a nonlinear system with static nonlinearity h, subject to the external wind disturbance g(V). It is emphasised that the function h is *independent* of the wind speed. This immediately suggests that the aerodynamic nonlinearity of the rotor can be accommodated by incorporating the inverse function h^{-1} within the controller (since h is independent of wind speed, implementing h^{-1} does not require any wind speed measurement). With the plant dynamics globally linearised in this way, the rest of the controller can be designed using linear methods. In fact, it can be shown that the conventional gain-scheduling approach for constant speed machines corresponds, albeit inadvertently, to precisely such a global inversion approach provided that an appropriate controller implementation is adopted (Leith & Leithead 1997).

Adopting a similar approach for variable speed machines, consider reformulating the nonlinear aerodynamics as

$$T(p,\Omega,V) = h(p,\Omega)-g(V) \qquad (3)$$

for some nonlinear functions h and g (figure 1c). Let $\hat{g}$ satisfy

$$\frac{d\hat{g}}{dV} = -\frac{\partial T}{\partial V}(p_{V_o}, \Omega_o, V_o) \qquad (4)$$

Note that (4) only defines $\hat{g}$ to within an arbitrary constant of integration but this does not affect the results which follow. Letting

$$\hat{h} = T(p,\Omega,V)+g(V) \qquad (5)$$

then it can be seen that when the aerodynamics are of the separable form, (3), $\hat{g}$ is equal (to within a constant) to g and $\hat{h}$ is independent of V and equal to h.

Of course, it remains to be established whether the aerodynamic characteristics of typical variable speed turbines do, in fact, exhibit the separable form, (3). Consider a typical 300 kW variable speed wind turbine previously studied in Connor & Leithead (1994) and which is representative of machines in its class. The aerodynamic characteristics of the rotor are illustrated graphically in figure 2. For this machine, $\hat{h}(p,\Omega)-\hat{g}(V)$ is plotted in figure 3 against $T(p,\Omega,V)$ (plots of the individual nonlinear functions $\hat{h}(p,\Omega)$ and $\hat{g}(V)$ are given in figure 4). It can be seen that in a large neighbourhood around rated conditions $\hat{h}(p,\Omega)-\hat{g}(V)$ is an accurate approximation to $T(p,\Omega,V)$. For example, when the pitch angle is 6 degrees the operating envelope over which $\hat{h}(p,\Omega)-\hat{g}(V)$ is an accurate approximation to $T(p,\Omega,V)$ is also marked explicitly by the dashed box in figure 2. This neighbourhood is considerably larger than the normal operating envelope of the turbine and so the representation can, for practical control design purposes, be considered globally valid. Similar results are obtained for a typical 1 MW variable speed wind turbine, although they are not included here owing to space limitations. It is stressed that (3) is valid non-locally in the sense that it is not confined to describing the behaviour about a single equilibrium operating point but rather describes the behaviour throughout the normal operating envelope.

4. GLOBAL LINEARISATION

Owing to the separability of the nonlinear aerodynamics, the compensation problem for variable speed wind turbines may be reformulated as one of linearising the memoryless nonlinearity, $\hat{h}(p,\Omega)$, whilst accommodating the dynamics of the actuator (which lie between the controller and the nonlinearity). It follows immediately that this is achieved by the approach depicted in figure 5a, where $\hat{h}^{-1}$, $\hat{A}_u$, $\hat{A}_u^{-1}$ are suitable approximations to, respectively, h^{-1}, the actuator dynamics and the inverse actuator dynamics. (Exact linearisation is achieved when there is no approximation error). Of course, $\hat{h}^{-1}$, must exist.

The strategy of figure 5a is, however, not unique. For example, from figure 5a,

$$\hat{p} = \hat{h}^{-1}(\hat{\phi},\Omega)$$

and, differentiating with respect to time,

$$\dot{\hat{p}} = \frac{1}{\frac{d\hat{h}}{d\hat{p}}(\hat{p},\Omega)}\left(\dot{\hat{\phi}} - \frac{d\hat{h}}{d\Omega}(\hat{p},\Omega)\dot{\Omega}\right) \quad (6)$$

Hence, it follows that figure 5a may be reformulated as depicted in figure 5b. (The differentiation operator in this formulation can be incorporated without difficulty into the linear controller owing to the integral action of the latter). Typically, the rotor speed varies slowly owing to the large inertia of the rotor in which case (6) can be simplified to

$$\dot{\hat{p}} = \frac{1}{\frac{d\hat{h}}{d\hat{p}}(\hat{p},\Omega)}\dot{\hat{\phi}} \quad (7)$$

It can be seen that the conventional gain-scheduling solution, in its simplified form without scheduling on $\partial T/\partial\Omega$, is closely related to the global linearisation solution, (7), provided an appropriate controller implementation is adopted. This is perhaps surprising since the gain-scheduling and global linearising solutions are arrived at by quite different approaches. In particular, the gain-scheduled solution is based on local linearisations and might otherwise be expected to only be valid near equilibrium operation while (7) is arrived at by nonlinear analysis and is effectively valid globally. One notable difference between the global solution, (7), and the gain-scheduled solution is that in (7) the control gain is scheduled with respect to both pitch angle and rotor speed while in gain-scheduling the gain is scheduled only with respect to pitch angle since the rotor speed is the same at every equilibrium point. This might be expected to lead to a difference in the performance achieved with the two approaches and this issue is currently under investigation.

5. CONCLUSIONS

It is shown that the nonlinear rotor aerodynamics of typical medium and large-scale variable speed wind turbines exhibit a separability property similar to that previously established for constant speed wind turbines. The importance of this result is that it establishes that from a control viewpoint a variable speed wind turbine can be viewed as a nonlinear system with static nonlinearity, h, subject to the external wind disturbance. It is emphasised that the function h is *independent* of the wind speed. This immediately suggests that the aerodynamic nonlinearity of the rotor can be accommodated by incorporating the inverse function h^{-1} within the controller (since h is independent of wind speed, implementing h^{-1} does not require any wind speed measurement). With the plant dynamics globally linearised in this way, the rest of the controller can be designed using linear methods.

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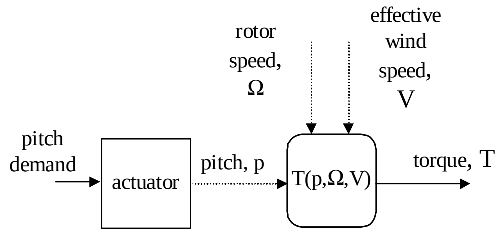

Figure 1a Dynamic relationship of aerodynamic torque to pitch angle and effective wind speed

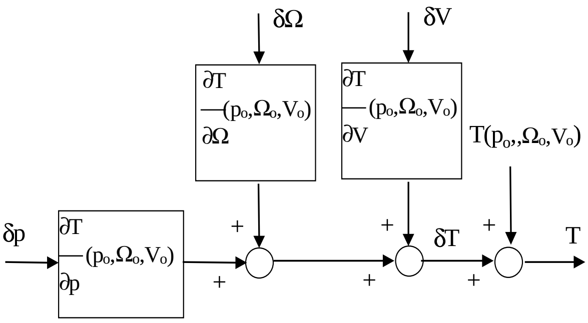

Figure 1b Local linearisation of aerodynamic non-linearity.

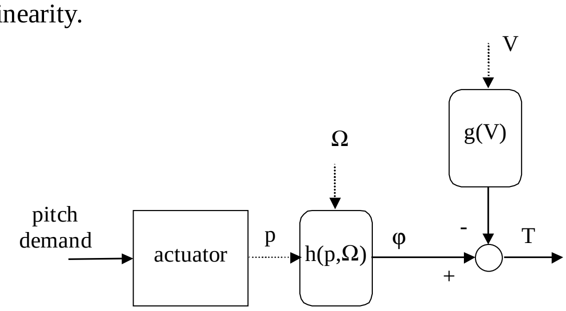

Figure 1c Reformulation (effectively global) of aerodynamic non-linearity in separable form.

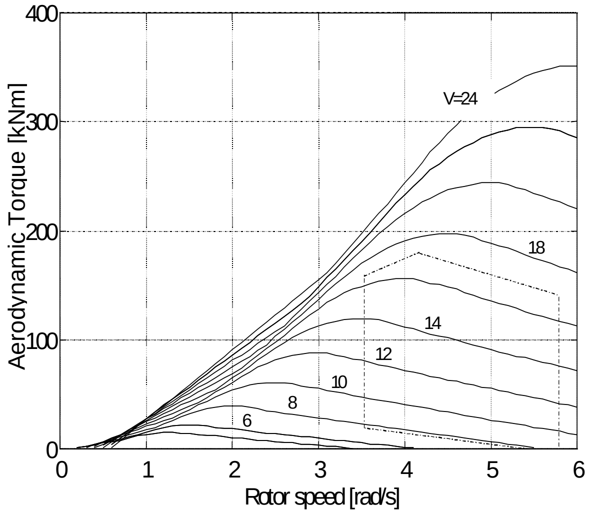

Figure 2 Aerodynamic torque versus rotor speed for 300 kW machine at pitch angle of 6 degrees. Rated torque is 71.75 kNm and rated speed is 4.64 rad/s

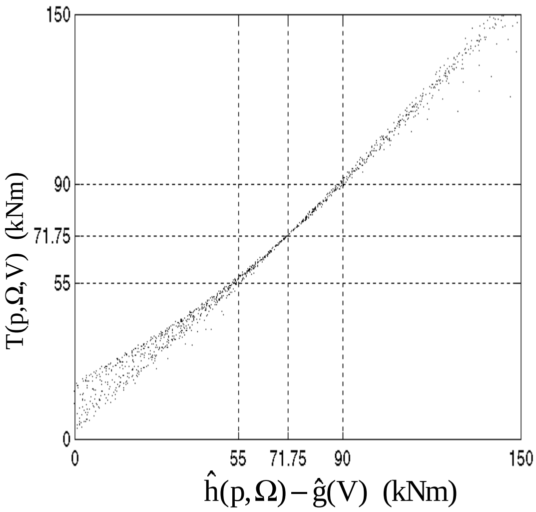

Figure 3 Torque vs $\hat{h}$ (p,Ω)- $\hat{g}$ (V) for 300kW machine. Rated torque is 71.75 kNm and rated speed is 4.64 rad/s.

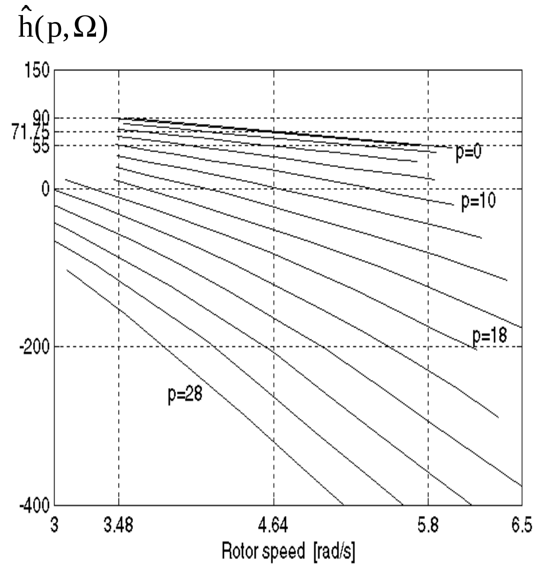

Figure 4a $\hat{h}$ (p,Ω) for 300kW machine

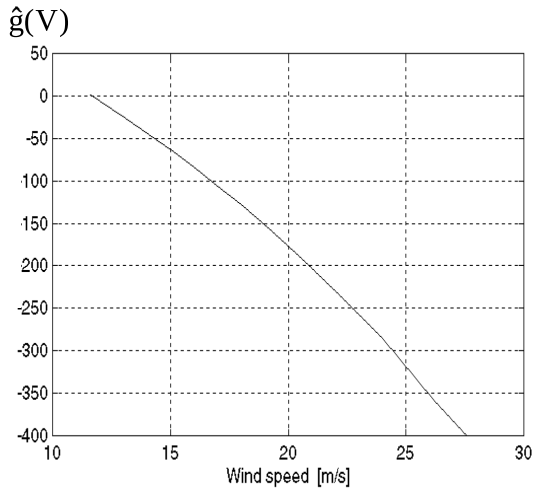

Figure 4b $\hat{g}$ (V) for 300 kW machine

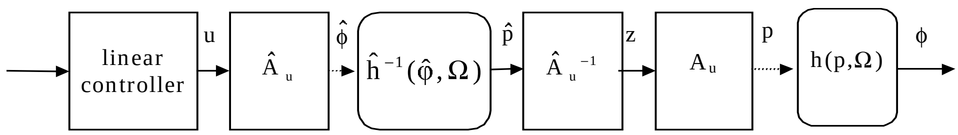

Figure 5a Direct linearisation

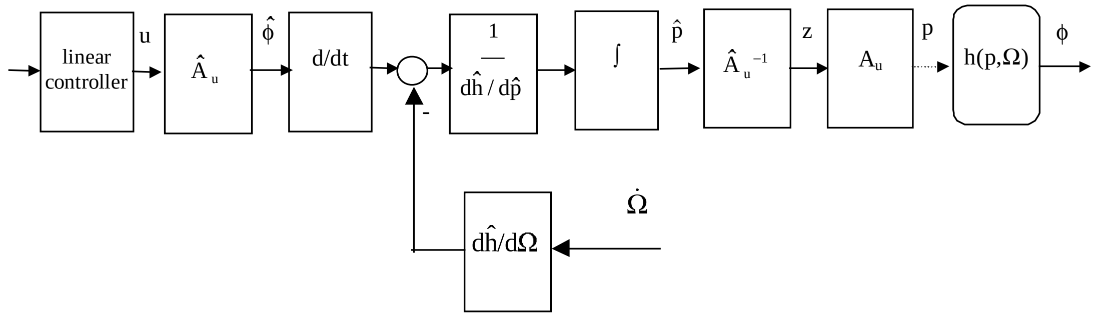

Figure 5b Direct linearisation with velocity form