COUNTER-EXAMPLE TO A COMMON LPV GAIN-SCHEDULING DESIGN APPROACH

D.J.Leith, W.E.Leithead

Department of Electronic & Electrical Engineering,
University of Strathclyde, Glasgow G1 1QE, U.K.
Tel. +44 141 548 2407, Fax. +44 141 548 4203 Email. doug@icu.strath.ac.uk

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Abstract

The shortcomings of a popular LPV gain-scheduling design approach are demonstrated by a simple counter-example. It is shown that, for a very general class of nonlinear systems, such an ad hoc design approach is unnecessary since soundly-based methods exist for transforming the plant dynamics into LPV/quasi-LPV form.

1. Introduction

Gain-scheduled controllers are generally linked by the design approach employed, whereby a nonlinear controller is constructed by interpolating, in some manner, between the members of a family of linear time-invariant controllers. In the conventional, and most common, gain-scheduling design approach, each linear controller is typically associated with a specific equilibrium operating point of the plant and is designed to ensure that, locally to the equilibrium operating point, the performance requirements are met. Under appropriate conditions (typically including some form of slow variation requirement) the stability of such a controller is guaranteed by long standing results (see, for example, Hoppensteadt 1966, Khalil & Kokotovic 1991).

Recently, a number of interesting alternative approaches have been proposed in the context of gain-scheduling design. Since these approaches employ various types of so-called linear parameter-varying (LPV) plant representation, they are commonly referred to as LPV gain-scheduling methods. The term "linear parameter-varying" is widely employed in the literature to refer to any system of the form $\dot{\mathbf{x}} = \mathbf{A}(\theta)\mathbf{x} + \mathbf{B}(\theta)\mathbf{r}$, $\mathbf{y} = \mathbf{C}(\theta)\mathbf{x} + \mathbf{D}(\theta)\mathbf{r}$ where θ is a parameter belonging to some class Ω. When θ is permitted to depend on the state, $\mathbf{x}$[1] (a situation widely considered in the literature, see examples in Shamma 1988, Shamma & Cloutier 1993, Coetsee 1994, Apkarian *et al.* 1995, Scherer *et al.* 1997, Apkarian & Adams 1998, Huzmezan & Maciejowski 1998, Lim & How 1998, Wu & Grigoriadis 1998, Johansen 1999) the dependence of the **A** and **B** matrices on the state introduces, of course, nonlinear feedback not present in linear time-invariant/time-varying systems. Use of the term *linear* parameter-varying to describe such nonlinear systems is, therefore, potentially misleading. In the context of the present paper, a generalisation of the terminology of Shamma (1988) is thus adopted and systems where θ may depend on the state, $\mathbf{x}$, are hereafter referred to as quasi-LPV systems while the term LPV is reserved for systems where θ is a strictly exogenous time-varying quantity (strictly independent of the state $\mathbf{x}$ of the system).

A considerable body of results now exists relating to the design of controllers for plants which are in LPV or quasi-LPV form. However, the literature typically takes the existence of a plant in LPV/quasi-LPV form as its starting point, largely neglecting the critical issue of how general nonlinear dynamics might be transformed to LPV/quasi-LPV form. Apparently lacking practical, generally applicable methods for carrying out such a transformation, a number of *ad hoc* approaches have been proposed in the literature. This paper considers one such popular approach whereby an LPV/quasi-LPV system is derived from the equilibrium linearisations of a nonlinear plant.

2. Design Approach & Counter-Example

Combining ideas from conventional and LPV/quasi-LPV gain-scheduling, the following hybrid control design procedure is similar to *ad hoc* approaches proposed in the literature (see, for example, Apkarian *et al.* 1995, Spillman *et al.* 1996, Fialho *et al.* 1997, Lee & Spillman 1997). Consider the nonlinear system,

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}, \mathbf{r}), \quad \mathbf{y} = \mathbf{G}(\mathbf{x}, \mathbf{r}) \qquad (1)$$

where $\mathbf{r} \in \Re^m$, $\mathbf{y} \in \Re^p$, $\mathbf{x} \in \Re^n$, $\mathbf{F}(\cdot,\cdot)$ and $\mathbf{G}(\cdot,\cdot)$ are differentiable. The series expansion linearisation of (1) about an equilibrium point $(\mathbf{x_o},\mathbf{r_o})$ is

$$\begin{aligned} \delta\dot{\mathbf{x}} &= \nabla_{\mathbf{x}}\mathbf{F}(\mathbf{x}_o,\mathbf{r}_o)\delta\mathbf{x} + \nabla_{\mathbf{r}}\mathbf{F}(\mathbf{x}_o,\mathbf{r}_o)\delta\mathbf{r} \\ \delta\mathbf{y} &= \nabla_{\mathbf{x}}\mathbf{G}(\mathbf{x}_o,\mathbf{r}_o)\delta\mathbf{x} + \nabla_{\mathbf{r}}\mathbf{G}(\mathbf{x}_o,\mathbf{r}_o)\delta\mathbf{r} \end{aligned} \qquad (2)$$

where

$$\delta\mathbf{x} = \mathbf{x}-\mathbf{x_0},\ \delta\mathbf{r} = \mathbf{r}-\mathbf{r_0},\ \delta\mathbf{y} = \mathbf{y}- \mathbf{G}(\mathbf{x_0}, \mathbf{r_o}) \qquad (3)$$

[1] It should be noted that, in order to apply LPV gain-scheduling methods in such circumstances, it is usually necessary to restrict the input and initial conditions of the state such that the solution $\mathbf{x}(t)$ is confined to some bounded operating region $X \subset \Re^n$ thereby ensuring that the "parameter" θ is bounded.

Assume that the locus of equilibrium operating points is parameterised by some quantity, $\theta(\mathbf{x},\mathbf{r})$. In order to transform the control design task into a form amenable to LPV/quasi-LPV methods, the following quasi-LPV system associated with the linearisation family defined by (2)-(3) is considered

$$\begin{aligned}\dot{\mathbf{z}} &= \mathbf{A}(\theta)\mathbf{z} + \mathbf{B}(\theta)\mathbf{r} \\ \mathbf{y} &= \mathbf{C}(\theta)\mathbf{z} + \mathbf{D}(\theta)\mathbf{r}\end{aligned} \tag{4}$$

where

$$\begin{aligned}\mathbf{A}(\theta) &= \nabla_{\mathbf{x}}\mathbf{F}(\theta(\mathbf{z},\mathbf{r})), \mathbf{B}(\theta) = \nabla_{\mathbf{r}}\mathbf{F}(\theta(\mathbf{z},\mathbf{r})) \\ \mathbf{C}(\theta) &= \nabla_{\mathbf{x}}\mathbf{G}(\theta(\mathbf{z},\mathbf{r})), \mathbf{D}(\theta) = \nabla_{\mathbf{r}}\mathbf{G}(\theta(\mathbf{z},\mathbf{r}))\end{aligned} \tag{5}$$

Standard LPV/quasi-LPV design methods can be applied to obtain a controller for the dynamics, (4)-(5). The controller obtained may then applied to the original nonlinear plant, (1).

Example

Consider the nonlinear system with dynamics described by

$$\begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-1 & 0 \\ 1 & 0\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix} + \begin{bmatrix}1 \\ 0\end{bmatrix} r + \begin{bmatrix}0 \\ -|x_2|x_2 - 10\end{bmatrix}, \quad y = \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix} \tag{6}$$

(It should be noted that systems with similar types of nonlinearity are frequently encountered in practice, see, for example, Nichols *et al.* 1993). The requirement is to design an output-feedback controller which ensures a step response settling time of less than 2 seconds with zero steady-state error (this is, of course, not a complete performance specification but is sufficient in the present context). The series expansion linearisation of (6) about an equilibrium point (x_{10}, x_{20}, r_o, y_o) is

$$\begin{bmatrix}\delta\dot{x}_1 \\ \delta\dot{x}_2\end{bmatrix} = \begin{bmatrix}-1 & 0 \\ 1 & -2|x_{2o}|\end{bmatrix}\begin{bmatrix}\delta x_1 \\ \delta x_2\end{bmatrix} + \begin{bmatrix}1 \\ 0\end{bmatrix}\delta r, \quad \delta y = \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}\delta x_1 \\ \delta x_2\end{bmatrix} \tag{7}$$

where

$$\delta x_1 = x_1 - x_{10},\ \delta x_2 = x_2 - x_{20},\ \delta r = r - r_0,\ \delta y = y - y_0 \tag{8}$$

The associated quasi-LPV system is

$$\begin{bmatrix}\dot{z}_1 \\ \dot{z}_2\end{bmatrix} = \mathbf{A}(\theta)\begin{bmatrix}z_1 \\ z_2\end{bmatrix} + \mathbf{B}r, \quad y = \mathbf{C}\begin{bmatrix}z_1 \\ z_2\end{bmatrix} \tag{9}$$

where

$$\mathbf{A}(\theta) = \begin{bmatrix}-1 & 0 \\ 1 & -2\theta\end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix}1 \\ 0\end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix}0 & 1\end{bmatrix} \tag{10}$$

and θ equals $|z_2|$. Assume, for the moment, that $0 \le \theta \le 10$ and θ may vary arbitrarily within this range (this assumption amounts to a restriction on the class of allowable initial conditions and inputs to the system such that $|z_2| \le 10$). Using standard software from the MATLAB LMI toolbox (Gahinet *et al.* 1995) and a conventional L_2 objective function with performance weighting, w_1, and control weighting, w_2, transfer functions[2]

$$w_1(s) = \frac{0.5}{s + 0.002}, \quad w_2(s) = \frac{2.0\text{x}10^{-2}s}{s + 1000} \tag{11}$$

the controller obtained for this system is

$$\dot{\mathbf{x}}_c = \mathbf{A}_c(\theta)\mathbf{x}_c + \mathbf{B}_c(y_{ref} - y), \quad r = \mathbf{C}_c\mathbf{x} \tag{12}$$

where $\mathbf{A_c}(\theta) = \alpha\mathbf{A}_o + (1-\alpha)\mathbf{A}_1$, $\alpha = (10-\theta)/10$,

$$\mathbf{A}_o = \begin{bmatrix} 5.6949e+003 & -2.7145e+000 & 2.7060e+001 & 7.6000e+004 \\ -3.5506e+004 & -1.5530e+001 & 2.4283e+002 & -4.7350e+005 \\ -1.9130e+005 & 2.8303e-001 & -5.6208e+000 & -2.5521e+006 \\ -2.0873e+004 & 3.1121e-001 & -1.6133e+000 & -2.7847e+005 \end{bmatrix}$$

$$\mathbf{A}_1 = \begin{bmatrix} 5.6777e+003 & -5.4523e-001 & 2.6029e+001 & 7.6032e+004 \\ -3.5504e+004 & -1.5777e+001 & 2.4295e+002 & -4.7350e+005 \\ -1.9130e+005 & 4.0036e-001 & -5.6766e+000 & -2.5521e+006 \\ -2.0869e+004 & -1.5741e-001 & -1.3905e+000 & -2.7848e+005 \end{bmatrix}$$

$$\mathbf{B}_c = \begin{bmatrix} 3.3565e-002 \\ -3.1706e-003 \\ -1.5648e-002 \\ 1.5715e+000 \end{bmatrix}, \mathbf{C}_c = \begin{bmatrix} 4.7249e+004 \\ 1.4691e+001 \\ -2.4353e+002 \\ 6.3016e+005 \end{bmatrix}^T \tag{13}$$

The response to step change in demand from –3.16 units to 0 units of the closed-loop system, consisting of the Jacobean-based quasi-LPV plant, (9)-(10), and controller, (12)-(13), is shown by the dashed-line in figure 1. (Note that θ (*i.e.* $|y|$) lies in the required range [0,10]). The settling time requirement is evidently satisfied. Nevertheless, when the same controller, (12)-(13), is applied to the original nonlinear system, (6), the corresponding response is as shown by the solid line in figure 1. It can be seen that the performance requirement is clearly not met and, indeed, that the nonlinear closed-loop system appears to be unstable.

Remark It is interesting to note that the performance requirement is in fact met for larger step demands; for example, a step change from –3.16 units to 6 units. This is perhaps unexpected, since larger steps are associated with excursions into operating regions further from equilibrium and with faster parameter variations, and clearly indicates that the behaviour observed is not associated with any restriction to near equilibrium operation arising from the use of equilibrium linearisations for the controller design. Indeed, the system in this example is intentionally selected to be benign in the sense that it satisfies the extended local linear equivalence condition of Leith & Leithead (1996,1998c); that is, the neighbourhood of validity of each equilibrium linearisation is unbounded and the union of these neighbourhoods covers the entire operating space. Hence, control design approaches based on the equilibrium linearisations are *not* a priori

[2] The performance requirement specifies zero steady-state error which implies that the magnitude of the transfer function of w_1 should be infinite at d.c. ; for example, by including an integrator term. However, the transfer functions here specify the actual values used in the numerical calculations, with approximate rather than exact integral action present in w_1.

restricted to near equilibrium operation. Further loss of performance associated with deviations from equilibrium operation can, of course, be anticipated for systems which do not satisfy such a condition.

■

The poor performance achieved in the foregoing example is perhaps unsurprising since no direct relationship is established between the quasi-LPV system, (9), used for control design and the nonlinear system which is actually of interest, (6). It is emphasised that the family of linear systems defined by the equilibrium linearisations of (6), being a *collection* of individual dynamic systems (each with its own distinct state, input and output defined by the transformations (8)) rather than a *single* dynamic system, is conceptually quite different from the quasi-LPV system, (9). Of course, controllers designed by approaches similar to that here may sometimes inadvertently achieve acceptable performance. Nevertheless, the foregoing example indicates that this is certainly not the case in general.

3. Velocity-based Design Approach

For a large class of nonlinear systems the foregoing *ad hoc* approach for transforming the plant dynamics into LPV/quasi-LPV form is unnecessary since soundly-based transformation methods exist. One such method considered here is based on the velocity-based framework recently developed by Leith & Leithead (1998a,b).

Before proceeding, it is useful to reformulate the nonlinear system, (1), as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{r} + \mathbf{f}(\rho), \quad \mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{r} + \mathbf{g}(\rho) \qquad (14)$$

where **A**, **B**, **C**, **D** are appropriately dimensioned constant matrices, $\mathbf{f}(\bullet)$ and $\mathbf{g}(\bullet)$ are nonlinear functions and $\rho(\mathbf{x},\mathbf{r})\in\Re^q$, $q\leq m+n$, embodies the nonlinear dependence of the dynamics on the state and input with $\nabla_{\mathbf{x}}\rho$, $\nabla_{\mathbf{r}}\rho$ functions of ρ alone. Trivially, this reformulation can always be achieved by letting $\rho = [\mathbf{x}^T\ \mathbf{r}^T]^T$, in which case q=m+n. However, the nonlinearity of the system is frequently dependent on only a subset of the states and inputs, in which case the dimension, q, of ρ is less than m+n. Since $\nabla_{\mathbf{x}}\rho$, $\nabla_{\mathbf{r}}\rho$ are functions of ρ alone, the variable, $\rho(\mathbf{x},\mathbf{r})$, equals the constant value, ρ_1, upon a surface of co-dimension q in Φ and $\nabla_{\mathbf{x}}\rho$ and $\nabla_{\mathbf{r}}\rho$ are constant over each surface. Hence, the normal to each surface is identical at every point on the surface and each surface is, therefore, affine. Moreover, to ensure that ρ is a unique function of **x** and **r**, these surfaces must be parallel for all ρ. Consequently, it may in fact be assumed, without loss of generality, that $\nabla_{\mathbf{x}}\rho$ and $\nabla_{\mathbf{r}}\rho$ are constant.

Differentiating (14), an alternative representation of the nonlinear system is

$$\dot{\rho} = \nabla_{\mathbf{x}}\rho\mathbf{w} + \nabla_{\mathbf{r}}\rho\ \dot{\mathbf{r}} \qquad (15)$$

$$\dot{\mathbf{w}} = (\mathbf{A}+\nabla\mathbf{f}(\rho)\ \nabla_{\mathbf{x}}\rho\ )\mathbf{w} + (\mathbf{B}+\nabla\mathbf{f}(\rho)\ \nabla_{\mathbf{r}}\rho\ )\dot{\mathbf{r}} \qquad (16)$$

$$\dot{\mathbf{y}} = (\mathbf{C}+\nabla\mathbf{g}(\rho)\ \nabla_{\mathbf{x}}\rho\ )\mathbf{w} + (\mathbf{D}+\nabla\mathbf{g}(\rho)\ \nabla_{\mathbf{r}}\rho\ )\dot{\mathbf{r}} \qquad (17)$$

Dynamically, (15)-(17), with appropriate initial conditions, and (1) are equivalent (have the same solution, **x**). While (14) and (18)-(20) are equivalent in the sense that they both embody the dynamics of the nonlinear system, they are not equivalent in other respects. In particular, the velocity representation, (15)-(17), may be trivially reformulated as the quasi-LPV system

$$\dot{\rho} = \nabla_{\mathbf{x}}\rho\mathbf{w} + \nabla_{\mathbf{r}}\rho\ \mathbf{z}, \quad \dot{\mathbf{r}} = \mathbf{z} \qquad (18)$$

$$\dot{\mathbf{w}} = (\mathbf{A}+\nabla\mathbf{f}(\rho)\ \nabla_{\mathbf{x}}\rho\ )\mathbf{w} + (\mathbf{B}+\nabla\mathbf{f}(\rho)\ \nabla_{\mathbf{r}}\rho\ )\mathbf{z} \qquad (19)$$

$$\dot{\mathbf{y}} = (\mathbf{C}+\nabla\mathbf{g}(\rho)\ \nabla_{\mathbf{x}}\rho\ )\mathbf{w} + (\mathbf{D}+\nabla\mathbf{g}(\rho)\ \nabla_{\mathbf{r}}\rho\ )\mathbf{z} \qquad (20)$$

where **z** is the input to the transformed system. Hence, it follows immediately that every nonlinear system, (14) , (and so every nonlinear system, (1)) can be rigorously reformulated as a quasi-LPV system, (18)-(20) to which the developed LPV/quasi-LPV control design methods may be brought to bear. When ρ depends only on the input, **r**, to the system (18)-(20) is an LPV rather than quasi-LPV system. When $\mathbf{w} = \mathbf{A}\mathbf{x}+\mathbf{B}\ \mathbf{r}+\mathbf{f}(\rho)$, $\mathbf{y} = \mathbf{C}\mathbf{x}+\mathbf{D}\mathbf{r}+\mathbf{g}(\rho)$ is invertible for every (**x**, **r**), so that **x** may be expressed as a function of **w**, **r** and **y**, then the transformation relating (18)-(20) to (14) is *algebraic*. The reformulation, (18)-(20), is clearly valid for a very general class of nonlinear systems. Moreover, it is emphasised that the velocity-based LPV/quasi-LPV representation is valid globally with no restriction whatsoever to a neighbourhood of the equilibrium operating points.

The relationship between (15)-(17) and (14) is evidently direct and it is argued that this directness is, in fact, a significant strength of this particular approach. Moreover, the directness of the relationship extends rather more deeply than might initially be expected. Consider the linear system, obtained by "freezing" (15)-(17) at an operating point at which ρ equals ρ_1,

$$\dot{\hat{\mathbf{w}}} = (\mathbf{A}+\nabla\mathbf{f}(\rho_1)\ \nabla_{\mathbf{x}}\rho\ )\hat{\mathbf{w}} + (\mathbf{B}+\nabla\mathbf{f}(\rho_1)\ \nabla_{\mathbf{r}}\rho\ )\dot{\mathbf{r}} \qquad (21)$$

$$\dot{\hat{\mathbf{y}}} = (\mathbf{C}+\nabla\mathbf{g}(\rho_1)\ \nabla_{\mathbf{x}}\rho\ )\hat{\mathbf{w}} + (\mathbf{D}+\nabla\mathbf{g}(\rho_1)\ \nabla_{\mathbf{r}}\rho\ )\dot{\mathbf{r}} \qquad (22)$$

The system (21)-(22) is referred to as the velocity-based linearisation of (14) associated with the operating point. It may be shown that when $\hat{\mathbf{w}}(t_1) = \mathbf{w}(t_1)$, $\hat{\mathbf{y}}(t_1) = \mathbf{y}(t_1)$ then the solutions to the linear system (21)-(22) are an accurate approximation to the solutions of the nonlinear system, (14), locally to the operating point (Leith & Leithead 1998a). Furthermore, while the solution to an individual velocity-based linearisation is only a locally accurate approximation, there exists a velocity-based linearisation, (21)-(22), for every operating point (**x**,**r**) and thus a velocity-based linearisation family, with members defined by (21)-(22), can be associated with the nonlinear system, (14). The solutions to the members of the family of velocity-based linearisations may be pieced together to approximate the solution to the nonlinear system (14) to an arbitrary degree of accuracy (Leith & Leithead 1998a). It is emphasised that, unlike conventional series expansion linearisation approaches, no restriction to near equilibrium operation is involved. This direct relationship between the nonlinear quasi-LPV systems and the linear system obtained by simply "freezing" the system at a particular parameter value is an important aspect of the velocity-based quasi-LPV formulation in the context of gain-scheduling.

Example (cont) The nonlinear system, (6), can be reformulated by differentiating, as

$$\dot{\rho}=w_2,\quad \begin{bmatrix}\dot{w}_1\\ \dot{w}_2\end{bmatrix}=\begin{bmatrix}-1 & 0\\ 1 & -2|\rho|\end{bmatrix}\begin{bmatrix}w_1\\ w_2\end{bmatrix}+\begin{bmatrix}1\\ 0\end{bmatrix}\dot{r},\quad \dot{y}=\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}w_1\\ w_2\end{bmatrix} \tag{23}$$

The system, (23), is in quasi-LPV form and existing quasi-LPV control design methods may be brought to bear. Of course, practical issues associated with the increased order of (23) in comparison to (6) and the presence of the derivative operator at the input to the reformulated plant remain to be adequately resolved. With regard to the order of the quasi-LPV representation, it is noted from (6) that

$$\begin{bmatrix}w_1-r\\ w_2\end{bmatrix}=T(x_1,x_2)=\begin{bmatrix}-1 & 0\\ 1 & -|x_2|\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} \tag{24}$$

Hence, the **x** and **w** states are related by a non-singular *algebraic* mapping with $x_2=T_2^{-1}(w_1-r,w_2)$. The velocity-based system, (23), may therefore be reformulated as the reduced-order system

$$\begin{bmatrix}\dot{w}_1\\ \dot{w}_2\end{bmatrix}=\begin{bmatrix}-1 & 0\\ 1 & -2|T_2^{-1}(w_1-r,w_2)|\end{bmatrix}\begin{bmatrix}w_1\\ w_2\end{bmatrix}+\begin{bmatrix}1\\ 0\end{bmatrix}\dot{r},\quad \dot{y}=\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}w_1\\ w_2\end{bmatrix} \tag{25}$$

With regard to the derivative operator at the plant input, it is noted that the steady-state performance specification requires that the controller contain pure integral action. Hence, by explicitly partitioning the controller into a pure integral term plus additional dynamics, say C_o, the velocity-based control loop depicted in figure 2a may, for design purposes, be reformulated as shown in figure 2b. The design task may now proceed by determining quasi-LPV controller dynamics, C_o, which achieve the required performance when applied with the reformulated plant augmented with an integrator at the output. The controller for the original nonlinear system, (6), is realised as the dynamics, C_o, followed by a pure integrator (see figure 2a). Using standard design software from the MATLAB LMI toolbox and an L_2 objective function with performance weighting w_1 and weighting w_2 on the control input r, as defined by (11), the controller dynamics, C_o, obtained for this system is

$$\dot{\mathbf{x}}_c=\mathbf{A}_c(\theta)\mathbf{x}_c+\mathbf{B}_c(y_{ref}-y),\quad r=\mathbf{C}_c(\theta)\mathbf{x} \tag{26}$$

where $\mathbf{A_c}(\theta)=\alpha\mathbf{A}_o+(1-\alpha)\mathbf{A}_1$, $\mathbf{C_c}(\theta)=\alpha\mathbf{C}_o+(1-\alpha)\mathbf{C}_1$, $\alpha=(10-\theta)/10$,

$$\mathbf{A}_o=\begin{bmatrix} 1.7058e+000 & 5.2402e-002 & -4.4428e-001 & 9.5434e-001 & 3.1461e+001\\ -2.3313e+004 & -2.3724e+001 & 3.6040e+003 & -1.0898e+002 & -4.0914e+005\\ 2.8731e+003 & 1.0562e+001 & -1.4210e+003 & 1.6039e+002 & 5.0429e+004\\ 7.6071e+002 & -1.3892e+000 & 2.9756e+001 & -1.8557e+001 & 1.3349e+004\\ -4.2678e+001 & 1.9619e-002 & -1.3072e+000 & -1.2245e-001 & -7.5074e+002 \end{bmatrix}$$

$$\mathbf{A}_1=\begin{bmatrix} 1.7056e+000 & -2.3312e+004 & 2.8729e+003 & 7.6071e+002 & -4.2678e+001\\ 5.6886e-002 & -5.5308e+001 & 1.4298e+001 & -1.3594e+000 & 1.9330e-002\\ -4.3108e-001 & 3.5110e+003 & -1.4100e+003 & 2.9842e+001 & -1.3079e+000\\ 1.0419e+000 & -7.2559e+002 & 2.3332e+002 & -1.7976e+001 & -1.2810e-001\\ 3.1461e+001 & -4.0914e+005 & 5.0429e+004 & 1.3349e+004 & -7.5074e+002 \end{bmatrix}$$

$$\mathbf{B}_c=\begin{bmatrix} 2.7342e-002\\ -1.5454e-005\\ -3.6988e-005\\ -2.8850e-003\\ -4.9850e+000 \end{bmatrix},\mathbf{C}_0=\begin{bmatrix} -2.3321e+004\\ -2.2619e+001\\ 3.5972e+003\\ -1.0787e+002\\ -4.0927e+005 \end{bmatrix}^T,\mathbf{C}_1=\begin{bmatrix} -2.3319e+004\\ -5.4161e+001\\ 3.5044e+003\\ -7.2368e+002\\ -4.0927e+005 \end{bmatrix}^T \tag{27}$$

The response of the nonlinear closed-loop system to a step change in demand from –3.16 units to 0 units is shown in figure 3. In contrast to the results obtained with an *ad hoc* quasi-LPV reformulation approach, it can be seen that the closed-loop system is stable and achieves the required performance (similar responses are also obtained for other magnitudes of step demand).

4. Summary

The shortcomings of a popular LPV gain-scheduling design approach are demonstrated by a simple counter-example. It is shown that, for a very general class of nonlinear systems, such an ad hoc design approach is unnecessary since soundly-based methods exist for transforming the plant dynamics into LPV/quasi-LPV form.

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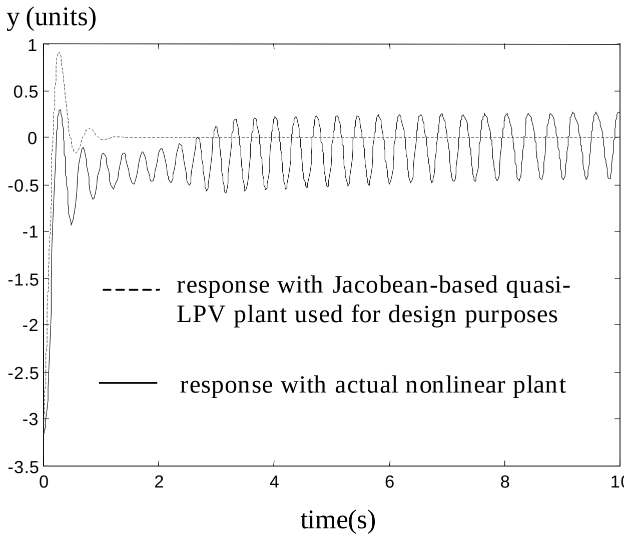

Figure 1 Nonlinear step responses for Jacobian-based quasi-LPV controller

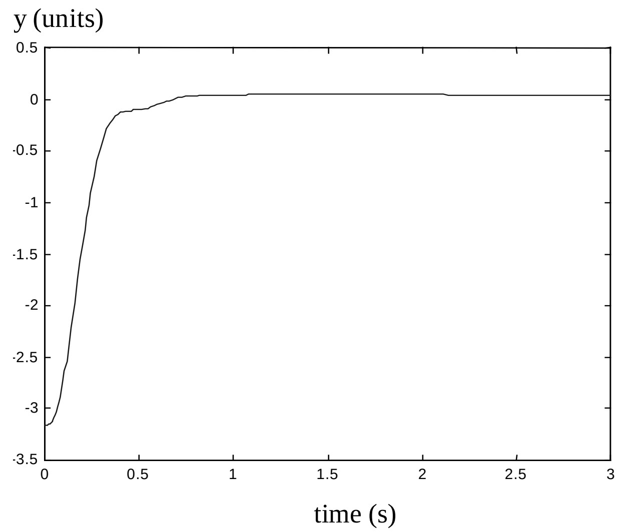

Figure 3 Nonlinear step response with velocity-based quasi-LPV controller

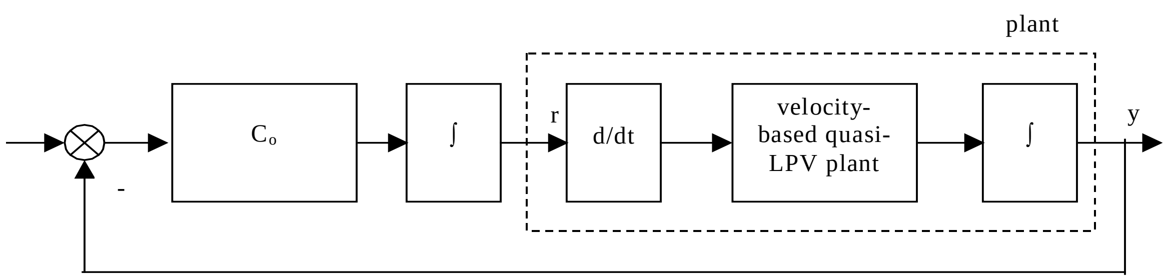

Figure 2a Velocity-based quasi-LPV control loop

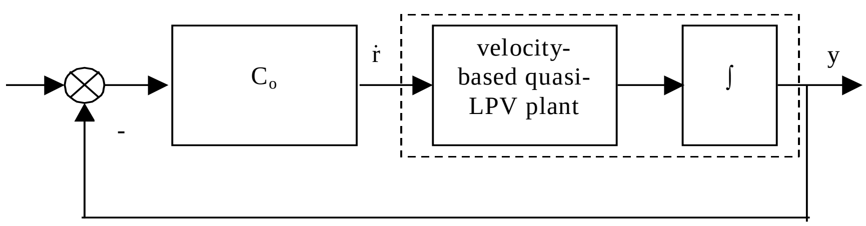

Figure 2b Reformulated velocity-based quasi-LPV control loop.