Sign Patterns that Allow a Positive Left Inverse

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Abstract

An $m \times n$ sign pattern $\mathcal{S}$ is an $m \times n$ matrix with entries in $\{+, -, 0\}$. Such a sign pattern allows a positive left inverse provided that there exist an $m \times n$ matrix A with the sign pattern $\mathcal{S}$ and an $n \times m$ matrix B with only positive entries satisfying $BA = I$ where I is the $n \times n$ identity matrix. For $m > n \geq 2$, a characterization of nonsquare sign patterns with no rows of zeros that allow a positive left inverse is given. This leads to a characterization of all sign patterns with $m \geq n \geq 2$ that allow a positive left inverse, giving a generalization of the known result for the square case, which involves the related bipartite digraph. Sign patterns that allow a nonnegative left inverse are also discussed.

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