Double Soules Pairs and Matching Soules Bases
by
Mei Q. Chen, Michael Neumann, Naomi Shaked-Monderer

Abstract

We consider two generalizations of the notion of a Soules basis matrix. A pair of nonsingular $n \times n$ matrices (P, Q) is called a *double Soules pair* if P and Q each has a positive first column, $PQ^T = I$ and $P\Lambda Q^T$ is nonnegative for every $n \times n$ nonnegative diagonal matrix Λ with nonincerasing diagonal elements. Chen, Han and Neumann gave an algorithm for generating such pairs. Here we characterize all double Soules pairs, and discuss some implications of the characterization. We also consider pairs (U, V) where $U \in \mathbb{R}^{m \times k}$ and $V \in \mathbb{R}^{n \times k}$ each has orthonormal columns, the first of which is positive, and $U\Lambda V^T$ is nonnegative for every $k \times k$ nonnegative diagonal matrix Λ with nonincerasing diagonal elements. We call such pairs *matching Soules pairs*. We characterize all such pairs, and make some observations regarding the nonnegative matrices generated by them.