

A Non-Linear Generalization of Perron-Frobenius Theory

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I will describe two classes of positive, monotonic mappings which have rich properties analogous in some ways to those of primitive matrices. The key enabling conditions are monotonicity and subhomogeneity, and the classes differ in the way that strictness appears in these conditions. The first class is primitive (i.e. "eventually strictly monotonic") and weakly subhomogeneous, while the second class is weakly monotonic but strictly subhomogeneous. Class I functions include primitive matrices and Morishima's non-linear homogeneous mappings, while class II mappings include "standard interference functions" that appear in Yates' power control work.

The two classes have similar properties, with Class II properties being generally stronger and easier to prove. I will talk about non-expansive and shrinking properties, fixed points, fixed directions, eigensolutions, and iterated mappings.