Nonhomogeneous Markov chains and web ranking

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PageRank is a popular algorithm for ranking the webpages. The score attributed to each webpages is derived from the stationary distribution of a random walk on the webgraph: it measures how often a given page is visited by a random walker on the web.

It seems realistic that this ranking may influence the reputation of the webpages, and therefore the behaviour of websurfers. We are interested in modelling the behaviour of a surfer who walks on the webgraph, favouring webpages with a high ranking. At each step of his walk, he moves to an out-neighbour j of his current page with probability proportional to $e^{E(r_j)/T}$, where r_j is the ranking of the webpage j, E is an increasing energy function and the temperature $T > 0$ is a fixed parameter measuring the confidence of the surfer in the ranking. The surfer's trajectory on the webgraph is then used in order to update the ranking of the webpages.

We study this model by considering the following two iterations on the set of stochastic row vectors

$$x_{t+1} = x_{t+1} M_T(x_t) \quad \text{and} \quad \tilde{x}_{t+1} = \tilde{x}_t M_T(\tilde{x}_t),$$

where the stochastic matrix $M_T(x)$ is a scaling of the webgraph's adjacency matrix C, which is supposed to be irreducible. We are interested in the fixed points and the convergence of both iterations, depending on the temperature T. When T is large enough, the fixed point of these iterations is unique and the convergence is global on the domain (for the second iteration, C is moreover supposed primitive). But for small values of T, at least when the matrix C is positive, there are always several fixed points.