Nonlinear Perron–Frobenius Theory

Ulrich Krause
Department of Mathematics, University of Bremen
krause@math.uni–bremen.de

Classical Perron–Frobenius Theory is about linear maps in finite dimensions which map the standard cone K into itself. The talk will present extensions of key theorems of the classical theory to nonlinear selfmappings of a given convex cone. An example is the following result.

Theorem T a concave selfmapping of K s.t.

- T maps each ray into a ray.
- T satisfies an indecomposability condition.
- For some unit vector $e_i \in K$ the i–th component of Te_i is (strictly) positive.

$\Rightarrow$

- The eigenvalue problem $Tx = \lambda x$ with $\lambda \in \mathbb{R},\ 0 \neq x \in K$ has a unique solution with $x = x^* > 0,\ \| x^* \| = 1,\ \lambda = \lambda^* > 0.$
- $\lim\limits_{k\to\infty} \frac{T^k x}{\|T^k x\|} = x^*$ for all $0 \neq x \in K$ ($\| \cdot \|$ any monotone norm).

This result covers not only nonnegative matrices which are indecomposable with a positive entry on the diagonal but also minima of affine–linear mappings as they occur in economics and maps with power–nonlinearities as they occur in biology. Similar results as the above theorem can be obtained for zigzag–operators which are not concave and map a cone of a Banach space into itself. Given the wide applicability of classical Perron–Frobenius Theory to various fields one can expect for a nonlinear extension an even richer body of applications.