Experiences with multi-path algorithms for joint routing and congestion control

Richard Gibbens

Computer Laboratory
University of Cambridge

Hamilton Institute Congestion Control Workshop
27–28 September 2005

Preamble

Talk arises from on-going collaboration with

- Frank Kelly
- Peter Key
- Tom Voice

Optimization problem

$$\max_{x_r, y_s \geq 0} \sum_{s \in S} U_s(y_s)$$

subject to

$$y_s^q = \sum_{r \in s} x_r^q \qquad \forall s \in S$$

and

$$\sum_{r : j \in r} x_r \leq C_j \qquad \forall j \in J$$

where $q = p/(p+1)$ for some $p > 0$.

Effect of q: valuing path diversity

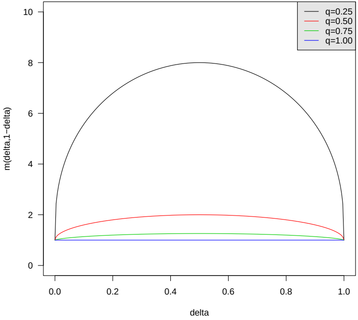
The quantity

$$m_q(x) = (\sum_i x_i^q)^{1/q}$$

is superadditive, that is,

$$m_q(x) + m_q(y) \leq m_q(x+y)$$

for all $x, y \geq 0$ and $0 < q \leq 1$. The graph shows $m_q((\delta, 1-\delta))$ for $0 < \delta < 1$ and varying values of q.



Choice of utility function

Assume

$$U_s(y) = w_s \frac{y^{1-\alpha}}{1-\alpha} \qquad \alpha \neq 1$$

and

$$U_s(y) = w_s \log(y) \qquad \alpha = 1$$

with $w_s > 0$ and $\alpha > 0$.

Reduced splitting dual algorithm

For each route r,

$$x_r(t) = \lambda_r(t)^{-(p+1)} w_{s(r)}^{p+1} y_{s(r)}(t)^{1-\alpha(p+1)}$$

where $\lambda_r(t) = \sum_{j \in r} \mu_j(t - T_{jr})$. For each link j,

$$\dot{\mu}_j(t) = \kappa_j \mu_j(t)(z_j(t) - C_j)^+_{\mu_j(t)}$$

where $z_j(t) = \sum_{r:j \in r} x_r(t - T_{rj})$. For each source s,

$$\dot{y}_s(t) = \kappa_s y_s(t)^{1/p} \left(\sum_{r \in s} x_r(t - T_r)^q - y_s(t)^q \right)^+_{y_s(t)} .$$

(cf. T. Voice (2005) *Stability of multi-path dual algorithms for joint routing and flow control*.)

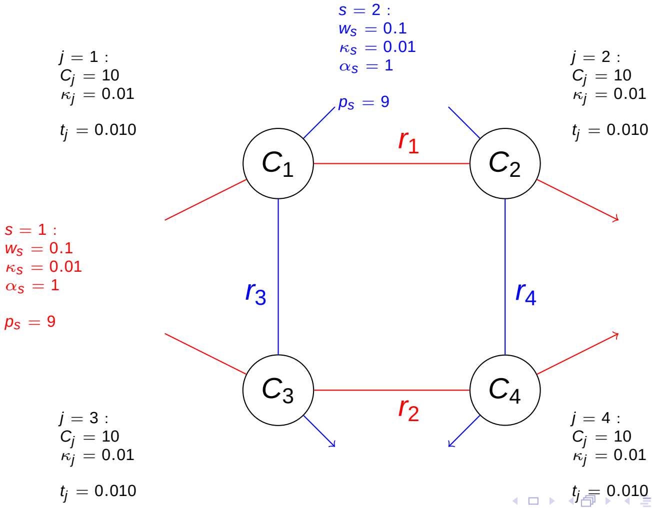
Network and user parameters

- sources, $s \in S$ comprising one or more routes $r \in R$
- routes, r comprising one or more resources, $j \in r$
- resource capacity $C_j, j \in J$
- delays $T_r = T_{rj} + T_{jr}$
- $q = p/(p+1)$
- $w_s > 0$
- $\alpha_s = 1$

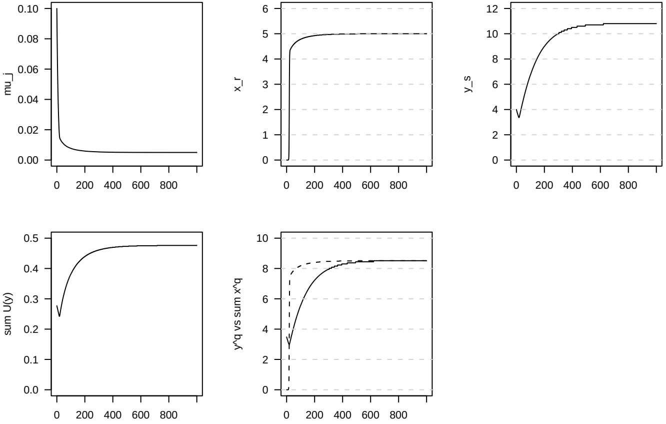
Algorithm parameters

- κ_j for each resource $j \in J$
- κ_s for each source $s \in S$
- initial prices, $\mu_j(0)$, for $j \in J$
- initial flows, $y_s(0)$, for $s \in S$

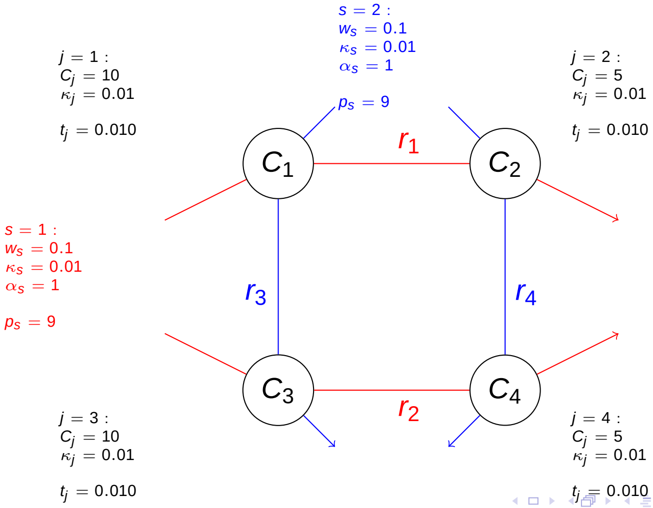
Example 4-node network — case 1



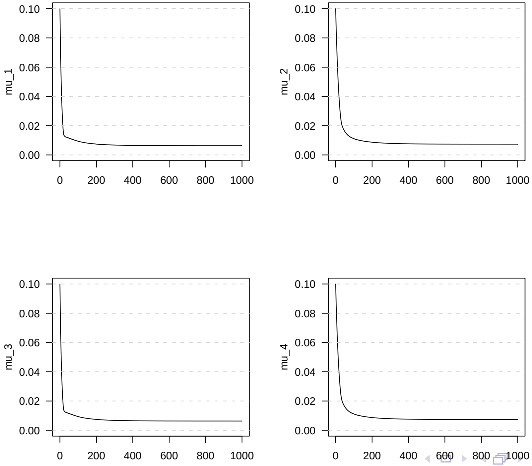
Results 1


mu_j
0.10
0.08
0.06
0.04
0.02
0.00
x_r
6
5
4
3
2
1
0
y_s
12
10
8
6
4
2
0
sum U(y)
0.5
0.4
0.3
0.2
0.1
0.0
y^q vs sum x^q
10
8
6
4
2
0
0
200
400
600
800

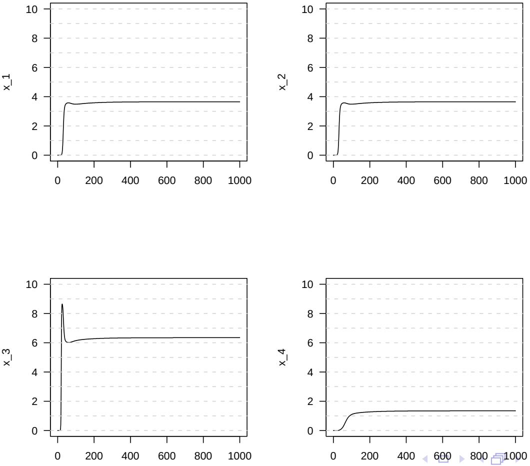
Example 4-node network — case 2



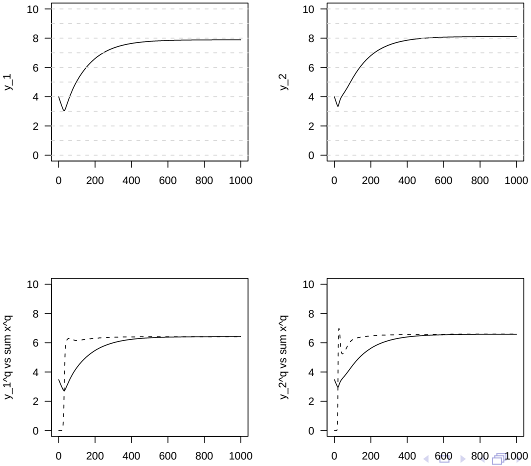
Results 2(a)



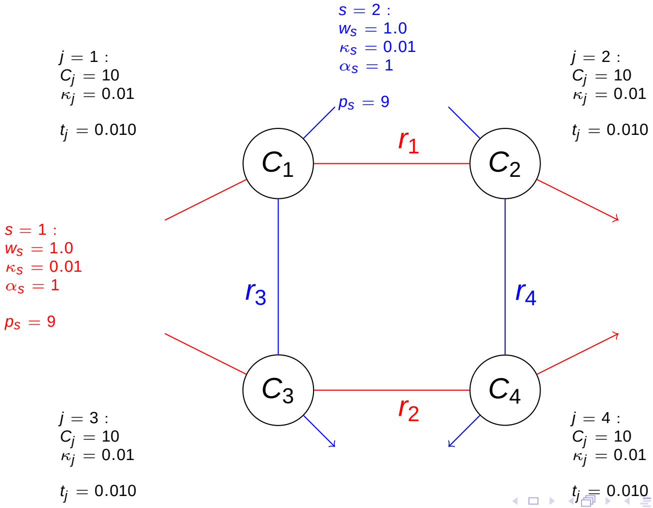
Results 2(b)



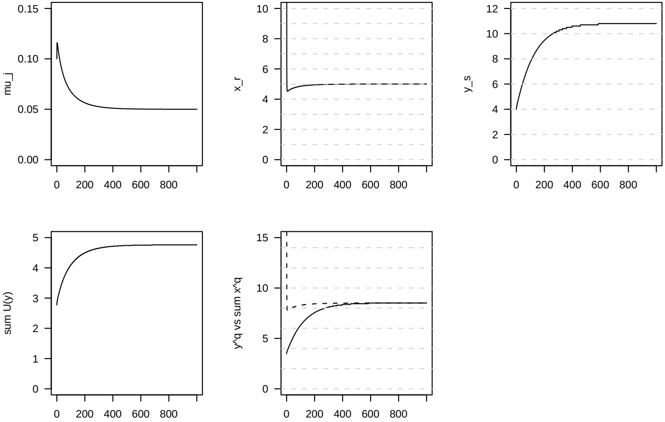
Results 2(c)



Example 4-node network — case 3



Results 3


mu_j
0.15
0.10
0.05
0.00
0
200
400
600
800
x_r
10
8
6
4
2
0
y_s
12
sum U(y)
5
4
3
2
1
y^q vs sum x^q
15
10
5

Extensions

Potential extensions to and connections with the following

1. Incentives for resource sharing in ad-hoc networks
2. Integration with wireless, not wired, networks
3. Adaptive route selection algorithms

Incentives for resource sharing

Build on approach taken in:

Jon Crowcroft, Richard Gibbens, Frank Kelly and Sven Östring
Modelling incentives for collaboration in mobile ad hoc networks
Performance Evaluation (2004) 57, 427–439

Credit transfers between nodes

- Arriving users join with unit credit
- Credit transfers determined by the congestion prices μ_j are exchanged between sources and the nodes used along routes
- Departing user takes their credit with them
- Concurrently, adjust each user's credit balance over time towards unity.

Cooperation provided by means of congestion fees paid by users from credit received from other users for forwarding via their own congested resources.

The form of capacity constraints in wireless networks

Mung Chiang
Balancing transport and physical layers in wireless multihop networks: jointly optimal congestion control and power control
IEEE JSAC, Vol 23, No 1, January 2005

$$\max_{x,P\geq 0} \sum_s U_s(x_s)$$

subject to

$$\sum x_s \leq c_\ell(P) \qquad \forall \ell$$

where

$$c_\ell(P) = \frac{1}{T}\log(1+K SIR_\ell(P)) \quad \text{and} \quad SIR_\ell(P) = \frac{P_\ell G_{\ell\ell}}{\sum_{k\neq\ell} P_k G_{\ell k} + \eta_\ell}$$

Adaptive route selection algorithms

Basic question

How do we dynamically adjust the active set of routes in the face of varying congestion and mobility?

Use the behaviour of the multi-path congestion/flow control algorithm as the feedback signal within a sticky random algorithm, much as for circuit-switched networks

R.J. Gibbens, F.P. Kelly and P.B. Key
Dynamic alternative routing — modelling and behaviour
In *Twelfth International Teletraffic Congress*. North-Holland (1988), Turin.

Summary

- Algorithms have been implemented and behaviour explored in example networks
- Helps build an understanding of the interactions between the many issues and parameters
- Potential extensions to:
  1. incentives for collaboration in mobile ad hoc networks
  2. alternative capacity constraints such as those for wireless
  3. algorithms for dynamic route set selection