Modeling 802.11 Mesh Networks

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***Abstract*— We introduce a tractable analytic model of throughput performance for general 802.11 multi-hop multi-radio networks subject to finite loads. The model's accuracy and utility is illustrated by comparison with simulation.**

I. Introduction

While there has been much recent progress on analytic modeling of single-hop 802.11 networks, the analysis of WLANs with more than one hop remains a challenging problem. In this paper we extend our finite load 802.11 single hop network model [1], [2] to introduce a tractable analytic model of throughput performance for 802.11 multi-hop networks. As far as we are aware, this is the first multi-hop analytic 802.11 model that supports finite loads and multi-radio multi-channel network topologies. Consideration of finite load is essential in mesh networks as, even if stations at the first stage in a relay network are saturated, losses at each relay imply that stations at subsequent stages need not be saturated. Thus, to determine scaling properties, finite load modeling is essential. Moreover, traffic such as voice and web is low-rate and network performance with such traffic cannot be modeled without consideration of finite loads. Consideration of multi-channel multi-radio WLANs is of considerable interest as such networks are becoming increasingly common and not only offer increased capacity but also have the potential to resolve fundamental issues such as hidden/exposed terminals.

II. Related Work

Most work on 802.11-based multi-hop networks focuses on issues such as routing and interference management (e.g. see [3], [4], [5], [6]) and changes to the 802.11 MAC to enhance performance (e.g. see [7], [8], [9]). Analytic modeling of the 802.11 CSMA/CA MAC in a multi-hop context has received relatively little attention. [10] considers the use of a single-hop saturated throughput model to support adaptive routing in multi-hop networks. [11] focuses on the saturated modeling of hidden station behavior in path and grid topologies. [12] considers a simplified throughput model in a random Poisson topology with saturated stations. In [13] a single hop model of 802.11 is introduced, though the authors comment it is only for valid light loads where achieved throughput is close to the offered load. They claim that this work can be extended to multi-hop networks by specifying the offered load at each stations but no analysis is presented. To the best of our knowledge, the present paper presents the first general 802.11 multi-hop model that supports finite loads and multi-radio multi-channel network topologies.

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III. Preliminaries

We make use of the following finite-load relationship in our multi-hop model. This relationship is derived in [1], [2] and the reader is referred there for further details. For each station we have a parameter q such that $1-q$ is the probability that the station's buffer has no packets awaiting transmission during a mean state time E (the medium can be in one of two states, idle or busy, and E is the mean state duration). For a station with given q and letting p denote the probability it experiences a collision conditioned on attempted transmission, from the operation of the 802.11 MAC the probability that the station is attempting transmission $\tau := \tau(p,q)$ is given by

$$\tau = \frac{1}{\eta}\left(\frac{q^2 W_0}{(1-q)(1-p)(1-(1-q)^{W_0})} - \frac{q^2(1-p)}{1-q}\right) \quad (1)$$

where

$$\begin{aligned}\eta &= \tfrac{qW_0}{1-(1-q)^{W_0}} + \tfrac{qW_0(qW_0+3q-2)}{2(1-q)(1-(1-q)^{W_0})} \\ &\quad +(1-q) + \tfrac{q(W_0+1)(p(1-q)-q(1-p)^2)}{2(1-q)} \\ &\quad +\tfrac{pq^2}{(1-q)2(1-p)}\left(\tfrac{W_0}{1-(1-q)^{W_0}} - (1-p)^2\right) \\ &\quad \left(\tfrac{2W_0(1-p-p(2p)^{m-1})}{(1-2p)}+1\right),\end{aligned}$$

W_0 is the station's minimum contention window and $W_0 2^m$ is the station's maximum window size.

The station's offered load Qkbps can be related to the probability q in a number of ways based on buffering assumptions. As we will use short interface buffers in the example of section V, here we just briefly explain the relation in that case. The reader is referred to [2] for further relations in other circumstances. The parameter q is the probability that at least one packet arrives in the expected time spent per state, E. The probability that at least one packet arrives during E is one minus the probability that the first inter-packet time is greater than E. Hence, when inter-packet arrival times are exponentially distributed with exponential rate λ we have that $q = 1 - \exp(-\lambda E)$. When a station is saturated, we take the limit as q tends to 1.

IV. Model

We wish to model an 802.11 mesh network. In particular a network with M distinct zones in which stations communicate locally on a common frequency that does not overlap with the frequencies used by neighboring zones. Within each zone we assume that there are no hidden stations and collisions only occur when more than one station attempts to use the medium. Certain stations in each zone are assumed to be equipped with multiple radios, but are not sources of traffic directly themselves. These stations can speak and hear in more than one zone and are used to relay traffic. We call stations that

have multiple radios *relay stations* and stations with only a single radio *local stations*.

For each zone $n \in \{1, \ldots, M\}$ we label local stations as elements of $\mathcal{L}_n = \{l_1^n, \ldots\}$ and relay stations as elements of $\mathcal{R}_n = \{r_0^n, \ldots\}$, allowing $\mathcal{L}_n$ or $\mathcal{R}_n$ to be empty. Consider a zone n. Time is slotted and we let τ_c denote the transmission probability of station c and p_c the corresponding collision probability. The collision probability p_c for each station $c \in \mathcal{R}_n \cup \mathcal{L}_n$ is determined by the following set of non-linear equations:

$$1 - p_c = \prod_{b \in \mathcal{R}_n \cup \mathcal{L}_n,\, b \neq c} (1 - \tau_b). \quad (2)$$

These equations state that the probability that station c does not experience a collision, given it is attempting transmission, is the probability that no other station within its zone is attempting transmission.

The medium in a zone can be in one of two states: idle or busy. The stationary probability of being idle is

$$p_{idle} = \prod_{b \in \mathcal{R}_n \cup \mathcal{L}_n} (1 - \tau_b).$$

The mean state length E_n in zone n is therefore $E_n = p_{idle}\sigma + L(1 - p_{idle})$, where each packet takes L seconds to be transmitted on the medium (we assume that collisions and successful transmissions both occupy L seconds) and the idle slot-length is σ seconds.

The achieved throughput S_c of station c is given by

$$S_c = \frac{\Lambda \tau_c (1 - p_c)}{E_n},$$

where Λ is the payload in bits and $\tau_c(1-p_c)$ is the probability station c does not experience a collision given it is attempting transmission (that is, the probability of a successful transmission).

It remains to determine the offered load at each station. Recall that we have two types of station: local and relay. The offered load Q_l at local station l arises from external traffic arrivals and is assumed to be known. However, the offered load at relay stations is determined by the manner in which traffic is routed between zones.

To obtain the offered load at relay stations we proceed as follows. We start by defining for each local station $l \in \mathcal{L}_n$, $n \in \{1, \ldots, M\}$, a fixed route f_l from its zone to a destination zone. Its route is an ordered set of relay stations through which l's packets must pass and a local station which is in the traffic's destination zone, with no relay repeated: $f_l = \{l, s_1 \ldots, s_m, d\}$, where d, a local station in the destination zone, is in the same zone as s_m. If $m = 0$, then l and d are in the same zone and no relaying is necessary. It is not important that we choose a specific destination as all stations within a zone hear all local transmissions. We assume routes are pre-determined by an appropriate routing protocol. Consider $l \in \mathcal{L}_n$ with route f_l. Let Q_{l,s_k} denote the offered load from l to relay station s_k. We assume that the proportion of traffic from l that makes it to s_k is a part of s_{k-1}'s throughput in the proportion $Q_{l,s_{k-1}}/Q_{s_{k-1}}$, where $Q_{s_{k-1}}$ is the overall offered load at s_{k-1}. We can then obtain the offered load on relay station s_k from

$$Q_{s_k} = \sum_{l \in \mathcal{L}_n, n \in \{1,\ldots,M\}, \{s_k, s_{k-1}\} \subset f_l} \frac{S_{s_{k-1}} Q_{l,s_{k-1}}}{Q_{s_{k-1}}}. \quad (3)$$

The network model is now complete. For given external loads $\{Q_l\}$ on each local station, we solve the non-linear equations in (1) and (2) for each zone, subject to the coupling constraint (3) being satisfied.

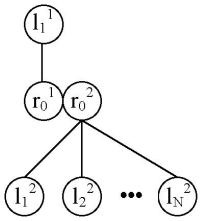
V. Example: relaying voice

We illustrate the model's validity in a simple scenario that highlights an important fairness issue that arises at aggregation points in 802.11 multi-hop networks. This issue has considerable impact on network performance and capacity, is a feature of the 802.11 CSMA contention mechanism and differs from fairness issues previously discussed in multi-hop wireless networks.

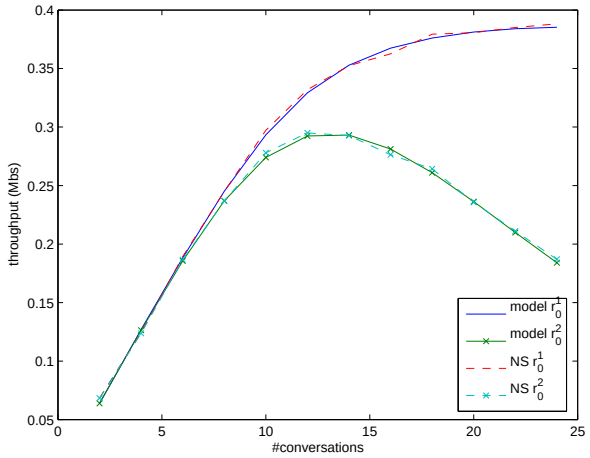
Consider the simple 802.11b multi-hop network depicted in Figure 1(a). The wireless station l_1^1 has a wired back-haul connection and communicates with the wireless clients $l_1^2, ...l_N^2$ via the wireless relay station r_0^1/r_0^2. The latter denotes a single relay station with two radios. We have $\mathcal{R}_1 = \{r_0^1\}$, $\mathcal{L}_1 = \{l_1^1\}$, $\mathcal{R}_2 = \{r_0^2\}$ and $\mathcal{L}_2 = \{l_1^2, \ldots, l_N^2\}$. The local stations in $\mathcal{L}_1$ and $\mathcal{L}_2$ communicate via the relays in $\mathcal{R}_1$ and $\mathcal{R}_2$. Thus we define the routes $f_{l_1^1} = \{l_1^1, r_1^1, l_1^2\}$ and, for each $n \in \{1, \ldots, N\}$, $f_{l_n^2} = \{l_n^2, r_1^2, l_1^1\}$. Note that any elements of zone 2 can be the recipient of l_1^1's traffic.

Suppose that the network carries two-way voice calls between client stations $l_i^2, i = 1, ..., N$ and back-haul gateway l_1^1. Voice calls are modeled as on-off 64Kbps traffic. Call parameters from [14]: two way on-off streams, the on period of an upstream call corresponds to the off period of its downstream reply, with exponentially distributed, mean 1.5 seconds, periods. The quantities of interest are the throughputs of the stations l_1^1 and l_n^2. In the model, we take each half of every conversation and treat it as a Poisson process with $Q_l = 32$kbps. Figure 1(b) compares throughput against number of active voice calls as predicted by the model, compared with NS packet-level simulation.

Observe that when more than eight voice calls active, throughput of the downstream calls begins to fall although upstream throughput continues to increase. It is this throttling of the downstream halves, rather than the physical radio bandwidth, that limits the network's voice call capacity. This occurs as 802.11's MAC layer contention mechanism allocates a roughly equal share of transmission opportunities to every wireless station. Thus client stations $l_i^2, i = 1, ..., N$ have roughly the same number of transmission opportunities as the relay station r_0^2. However, the relay station is required to transmit the downstream part of N voice calls whereas each client station only transmits the upstream part of a single voice call. The model's accuracy in this scenario gives great promise and reflects the precision we have seen in other setups not reported on here due to space constraints.



(a) Topology



(b) Throughput

Fig. 1. Mesh WLAN topology in Figure 1(a). Voice packets are transported between l_1^1 and $l_1^2, ..., l_N^2$ by station r_0^1/r_0^2 which denotes a relay station with two radios. Model predictions and NS simulation.

VI. Model scope

We assumed a fixed packet size because of space constraints. This assumption can be relaxed, but one must then keep track of the packet-size distribution in each zone. We have used our equation (1) to relate p, q and τ, but any other relation of this sort could be used instead. For example, if stations are saturated, then the classical Bianchi [15] model can be used. However, in a multi-hop context we cannot generally assume that relay stations are saturated even when all local stations are saturated. Even with simple topologies, losses at upstream relay stages mean that downstream relays need not be saturated. Hence it is almost always necessary to use a model that captures finite load effects. It would also be possible to use relations from multi-class 802.11e models, such as those presented in [16]. For notational convenience we have assumed relay stations do not generate traffic themselves, but this can be readily included. Our model does not treat hidden nodes. The impact of hidden terminals could be included using a similar approach to [17], although we have not done this here. Our main interest in this paper is in multi-radio settings where channels are chosen so that hidden/exposed terminals are avoided.

VII. Conclusions

We have introduced a remarkably accurate tractable analytic model of throughput performance for general 802.11 multi-hop networks and presented an example to illustrate its use. As future work we plan to use the model to determine scaling behavior of throughout with number of relay stages, examine other performance anomalies of multi-hop 802.11 and their possible correction using the flexibility of 802.11e.

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