Using a Car Sharing Model to Alleviate Electric Vehicle Range Anxiety

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Abstract—A method is proposed to reduce range anxiety problems for electric vehicle owners using a car sharing model. Basing the model on elementary models from queueing theory, it is argued that the potential cost of such a system would be much less than the current government subsidies offered to support the deployment of electric vehicles.

I. INTRODUCTION

Growing concerns over the limited supply of fossil-based fuels are motivating intense activity in the search for alternative road transportation propulsion systems. Regulatory pressures to reduce urban pollution, CO₂ emissions and city noise have made zero-emission (ie: fully battery powered) electric vehicles (EVs) and plug-in hybrid vehicles (PHEVs) [1] a very attractive choice as the alternative to the internal combustion engine (ICE) [2]. Road transportation currently accounts for 22-25% of emissions in the UK [3]-[7], for example, and thus represents a significant contribution to global warming in the UK. Such figures are typical of most western countries, and electric vehicles, which have zero emissions of pollutants in urban areas, are seen as a useful tool both in reducing urban pollution, and in reducing the carbon footprint of road transportation generally (when combined with clean generation), in addition to reducing our dependency on fossil fuels.

While the environmental and societal benefits of zeroemission vehicles are evident, their adoption by users has been extremely disappointing. According to recent reports [8], even in Europe, where the green agenda is well received, fewer than 12000 electric vehicles were sold in the first half of 2012 (of which only 1000 of these were sold in the UK). This number represents less than 0.15% of total new car sales. Figures such as these are nothing short of disastrous for companies such as Renault or Nissan, both of whom have placed massive bets on widespread adoption of electric vehicles [9]. These figures are in spite of the fact that many European governments have offered incentives for the purchase of electric vehicles in the form of subsidies and have also invested massively in infrastructure. For example, in Portugal, the government installed 1300 charging spots

F. Wirth is with the Department of Mathematics, University of Würzburg, Am Hubland, 97074 Würzburg, Germany. in a number of cities, and offered subsidies of \$5000 to encourage the purchase of such vehicles [10].

The key factors hindering the widespread adoption by the general public of electric vehicles may be summarised briefly as follows:

(i) Cost and availability of rare earth materials - Plugin vehicles tend to be very expensive, even when subsidised. A major factor in the cost of such vehicles is the cost of the battery [11]. Lithium-based batteries are expensive, and while costs are forecast to reduce dramatically over the next few years [12], this factor is an important one in understanding the sales of electric vehicles at the present time. In response to this, Renault, and other companies, are proposing to lease batteries to the customer to offset some of the battery related costs. However, even if such initiatives are successful, there are other battery related questions that may hinder the adoption of electric vehicles. A fundamental question concerns whether enough lithium can be sourced to build batteries to construct enough vehicles to replace the existing passenger vehicle fleet. Are we simply substituting one rare resource (oil) with another (lithium)? Also, the transportation of batteries is not trivial and necessitates special precautions. This may be an issue that needs attention before widespread adoption is possible [13].

(ii) **Electromagnetic emissions** – A recent issue regarding electric vehicles concerns electromagnetic emissions. While there is no evidence that EM radiation from EVs is dangerous, this issue is a focus point for regulatory authorities (see EU Green Car Programme) and has been raised by several research agencies [14].

(iii) **Long charging times** – One issue that is deserving of special attention is that of vehicle charging. Charging times for electric vehicles are known to be long [15]. An often cited fact by advocates of electric vehicles in response to this is that fast charging algorithms can service average vehicles in about 30 minutes [16]. Such time-scales may be just about acceptable to a normal car owner. However, in the presence of queuing, 30 minutes can rapidly become several hours, and push such fast charging stations into the realm of "not acceptable." Thus, it is likely that overnight or workplace charging will be the principal method of vehicle charging for the foreseeable future. An associated issue in large cities concerns the availability of charging points. This is especially an issue in cities with large apartment block type dwellings.

(iv) **Vehicle size** – Electric vehicles tend to be designed small with limited luggage space to reduce energy consump-

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tion. This is a significant problem for most potential purchasers of vehicles who on occasion would like to transport significant loads using their vehicles.

(v) **Range anxiety** – One of the most pressing issues in the deployment of EVs concerns the issue of range anxiety [2], [11], [17]. Maximum ranges (in favourable conditions) of less than 150km are not unusual for electric vehicles, and this reduces significantly when air-conditioning or heating is switched on. The issue of limited range also exacerbates other issues. For example, the cost of searching for a parking position at the end of a journey is much higher than for a conventional vehicle (because the EV's range is so low and therefore energy should not be wasted searching for a parking spot). Research is ongoing to address these issues, with much of the current work focussing on new battery types, optimal vehicle charging, vehicle routing, and in-vehicle energy management systems with a view to minimising wastage of energy and thereby increasing vehicle range [18].

While all of the above issues are important, and some others that we have not mentioned, such as the design of vehicles with in-wheel motors, in this paper, we focus on the pressing issue of consumer range anxiety, and to a lesser extent, vehicle size. A solution is proposed to these problems based on a car sharing concept along the lines of car-to-go [19] and drive-now [20]. In Section II, we present a high-level overview of the proposed car sharing scheme, and pose a number of Quality of Service (QoS) questions. Using elementary probability and queueing theory methods, solutions to the Quality of Service (QoS) questions are formulated mathematically in Section III. Section IV illustrates by example how the cost of the proposed car sharing system compares to current government subsidies offered to support the deployment of electric vehicles. Directions for future work are provided in Section V.

II. HIGH-LEVEL DESCRIPTION

The objective is to propose a car sharing concept as a solution to the consumer range anxiety problem. The proposed car sharing concept is as follows.

When an electric vehicle is purchased, the new EV owner also automatically becomes a member of a car sharing scheme along the lines of car2go or DriveNow [19], [20], where a shared vehicle may be borrowed from a common pool on a 24hr basis. The shared vehicles are large ICEbased vehicles suitable for long range travel and with large goods transportation capacity.

Remark 1: We suggest free membership of the scheme, but a pricing model could be implemented to regulate demand on weekends, public holidays, or other occasions when synchronised (correlated) demand is likely to emerge, or to regulate emissions.

Remark 2: If the shared ICE-based vehicles are chosen to be sufficiently high-end, then a further incentive for consumers to purchase electric vehicles is provided.

Remark 3: What we are proposing is nothing more than a hybrid vehicle with a temporal component.

A number of issues would need to be resolved before any such system could be deployed. These issues fundamentally reduce to the marginal cost of the system. More specifically, we wish to determine if such a sharing concept could be deployed giving reasonable quality of service (QoS) to the electric vehicle owner, without significantly increasing the cost of each vehicle. Referring to Figure 1, this amounts to asking whether a reasonable QoS can be delivered when M, the number of shared ICE-based vehicles, is significantly less than N, the number of purchased EVs. To answer this question, we consider two scenarios (both under the assumption that a shared vehicle is borrowed for a 24hr period).



Fig. 1. Car sharing concept.

<u>Problem 1</u>: Spontaneous journeys – Given that an individual requests a vehicle, what is the fraction $\frac{M}{N}$ corresponding to a fixed (low) probability that the request is declined?

Problem 2: Planned journeys – For a fixed M and N, how many days in advance does a user have to make a reservation so that the probability that the request is declined is lower than some very small constant? This gives rise to a queuing model.

The following demographical assumptions will be made:

- long journeys in private cars are rare, meaning that the range of an electric vehicle, even under worst-case conditions (eg: air conditioning use, traffic congestion, bad weather), should be sufficient for most journeys;
- most private vehicles are in use¹ for an entire "waking hour" day, and most urban dwellings are houses rather than apartments, meaning that there is no structural impediments to overnight charging, and that a full overnight charge should be sufficient to satisfy the needs of daily mobility patterns.

Contemporary Irish mobility patterns, for example, align with the above assumptions. Data for the creation of Table I and Figures 2, 3 and 4 were obtained from the 2009 Irish National Travel Survey (NTS) Microdata File, Central Statistics Office, ©Government of Ireland [21]–[23]. In the NTS, respondents were asked to provide details about their

¹*In use* meaning that a private vehicle is not parked at home for a sufficient period to fully charge.

 TABLE I

 Percentages of people who drove cumulative distances of greater than 50km, 75km and 100km over a 24h period.

Sample Population	>50km	>75km	>100km
Monday	23%	12%	7%
Tuesday	23%	14%	8%
Wednesday	23%	14%	7%
Thursday	26%	18%	11%
Friday	26%	17%	9%
Saturday	24%	15%	9%
Sunday	24%	17%	11%

travel for a given (randomly selected) 24h period, which roughly corresponded to a day of the week. Table 1 shows the percentages of people who drove (over the 24h period that they were queried about) cumulative daily distances of greater than 50km, 75km and 100km. Figure 2 relates to those people who were questioned about their travel over the 24h "Monday" period (ie: row two of Table I), and depicts number of people versus total distances they drove over that 24h Monday period. Figure 2 illustrates a trend observed in the percentages in Table I; namely, that longer cumulative journeys over the course of a day were rare. (Graphs of the nature of Figure 2, but concerning travel for the other days of the week, were similar in shape to Figure 2, and have thus been omitted to reduce redundancy.) For respondents who drove cumulative distances greater than 75km over a 24h period (see the third column of Table I), Figure 3 illustrates the hours of a 24h period over which respondents had their vehicles in use (many vehicles were in use roughly between 8am and 6pm), and Figure 4 depicts the number of respondents versus total time (out of a 24h period) their vehicle was in use.



Fig. 2. Number of survey respondents reporting about their travel for the 24h "Monday" period, versus total distances they drove over that period.

III. MATHEMATICS

Using elementary probability and queueing theory methods, we now pose solutions to the Quality of Service (QoS)



Fig. 3. Number of respondents (who drove >75km total daily distance) using their vehicles, versus hour of the day.



Fig. 4. Number of respondents (who drove >75km total daily distance), versus total time that their vehicle was in use for over the 24h survey period.

problems presented in Section II. Consider a population of N electric vehicle owners (ie: N "users") who occasionally require access to an ICE-based vehicle (ICEV) for a non-standard trip (either a long-range trip or a trip where large load carrying capacity is required). We assume that a user will keep the ICEV for a full day, based on the driver behaviours described in the previous section. Thus, each day is characterised by the number of users who require an ICEV on that day. There is a fleet of M ICEVs available to satisfy this need. The main question is then to determine the relation between M and N. This will be determined by requiring some quality of service (QoS) conditions to be met. For example, a QoS condition might be a guarantee on the probability of finding an ICEV available.

A. Model 1 – Binomial Distribution

In the simplest model, each user independently requests an ICEV each day with a fixed probability p. The probability can be estimated from the data presented in Table I. Thus the number of requests X each day is a binomial random

variable:

$$X \sim \operatorname{Bin}(N, p)$$

The mean number of requests per day is Np, and the standard deviation is $\sqrt{Np(1-p)}$. In principle, the number of requests may be anything from 0 to N, but for large N it is very unlikely that X will deviate from the mean by more than a few standard deviations.

This QoS condition can be quantified as follows. For each $M \leq N$, define

$$Q(M) = P(X > M)$$

Then the QoS condition could be to find the smallest M such that $Q(M) < \varepsilon$ for some specified ε . For any given N and p this can be calculated explicitly using the formula

$$Q(M) = \sum_{k=M+1}^{N} {\binom{N}{k}} p^{k} (1-p)^{N-k}$$

However it is more useful to get an approximate formula from which the scaling relation can be read off. For N large enough we can use the normal approximation for the binomial, which says that

$$Z = \frac{X - Np}{\sqrt{Np(1 - p)}}$$

is approximately a standard normal random variable. There is a standard rule of thumb regarding applicability of the normal approximation for the binomial, namely that

$$N \ge 9 \max\left(\frac{p}{1-p}, \frac{1-p}{p}\right)$$

Using the normal approximation, we have

$$Q(M) \simeq rac{1}{\sqrt{2\pi}} \int_r^\infty e^{-rac{1}{2}X^2} dX, \qquad r = rac{M - Np}{\sqrt{Np(1-p)}}$$

This readily yields estimates for M in order to satisfy a desired QoS condition. For example, in order to satisfy the QoS condition

meaning a less than 5% chance of not finding an ICEV available, it is sufficient to take

$$r \ge 1.65 \iff M \ge Np + 1.65\sqrt{Np(1-p)}$$

(see Figure 5). For example, using the values N = 1000 users and p = 0.1 for the probability of a user requesting an ICEV, this provides a value $M \ge 116$.

B. Model 2 – A Queueing Model

When the number of requests exceeds the number of available ICEVs, a queue forms and users must wait one or several days until a vehicle becomes available. It is desirable to keep the probability of long delays small, and this can be achieved by appropriate scaling of M with N.

Let X_n be the number of outstanding requests at the end of the n^{th} day, and let A_n be the number of new requests that



Fig. 5. Number of required extra vehicles for 5% chance of not finding a vehicle

arrive during the n^{th} day. Since the number of vehicles is M, the relation between these variables on successive days is

$$X_{n+1} = \max\{0, X_n - M\} + A_{n+1}$$

That is, the queue length is reduced by M at the start of each day (but not reduced below zero), and is then increased during the day by the number of new requests.

The QoS condition is to ensure that X_n is unlikely to be large, implying that users are unlikely to have to wait a long time before being assigned an ICEV. Since *M* users can be serviced each day, in the worst-case a user must wait $\lfloor X_n/M \rfloor$ extra days until service. We consider the QoS condition which guarantees that the probability that any user needs to wait *k* extra days or more is less than ε , that is

$$P(X_n > kM) < \varepsilon$$

By choosing M sufficiently large we can guarantee that this probability is small. (Due to space constraints, the proof to the following lemma will appear elsewhere.)

Lemma 1: Define

$$\mu = M - Np, \qquad \sigma^2 = Np(1-p), \qquad \alpha = \frac{\mu}{\sigma^2}$$
Then for all $k \ge 1$,

$$P(X_n > kM) \le \frac{1}{2}e^{-(k-1)M\alpha} \left(e^{\mu\alpha/2} - 1\right)^{-1}$$

Using the bound in Lemma 1 we find a sufficient condition to guarantee the QoS bound, namely

$$\frac{1}{2}e^{-(k-1)M\alpha}\left(e^{\mu\alpha/2}-1\right)^{-1}<\varepsilon$$

For a given k and ε we may use this to find a value for M needed to meet the QoS condition. For example, using the same values as above N = 1000, p = 0.1, $\varepsilon = 0.05$, and taking k = 3 (meaning we want 95% of users to receive an ICEV

within 4 days or less), we find that the inequality is satisfied whenever $M \ge 101$. Taking k = 2 we find $M \ge 105$, and with k = 1 we find $M \ge 121$. For a fleet size of 20000 vehicles with M = 2000 (10%) and M = 3500 (17%), the behaviour of the bound is depicted in Figures 6 and 7, respectively, for various estimates of the probability of a long distance trip. As can be seen, the bound tends rapidly to zero, indicating that the probability of waiting for a shared vehicle longer than one or two days vanishes rapidly.



Fig. 6. Probability (bound) of not finding a car with N = 20000 and M = 2000 (10%).



Fig. 7. Probability (bound) of not finding a car with N = 20000 and M = 3500 (17%).

IV. RESULTS

We now explore the financial cost of setting up the car sharing model. We ignore, for now, any costs associated with the management of the car sharing concept, or the costs of maintenance or replenishment of the shared fleet. We assume a once-off purchase of the shared fleet. In reality, the shared cars would be replaced after a fixed period, thereby reducing the cost of the shared vehicles even further. For the purpose of benchmarking, we assume that our shared fleet is constructed to reflect the needs of the EV owners; namely, sometimes a family car is needed, sometimes a large vehicle is needed for transporting goods, and sometimes a smaller car is required for short out of town trips by a single person/couple.

We use Volkswagen vehicles to benchmark the cost of the shared vehicle fleet. Note VW is a relatively high-end marque; and the vehicle fleet could be constructed in a manner that is considerably cheaper. Table II illustrates the costs involved in a fleet composed in this manner, with all costs sourced from the VW website (www.vw.ie) as on 25th September, 2012. The average vehicle cost is thus €23376.

Purpose	Percentage	Sample Vehicle	Cost
Singles/Couples	20%	VW Polo	€15345
Family without luggage	20%	VW Golf	€21180
Family with luggage	50%	VW Passat	€26245
Transport	10%	VW Passat Estate	€29480

TABLE II

COMPOSITION OF SHARED VEHICLE FLEET.

We also assume a fleet of Nissan Leaf electric vehicles. These electric vehicles currently retail for \in 35000 in Ireland (25/9/2012).

For convenience, we recall Table I. Table I indicates that the probability of journeys greater than 75 km is approximately 0.15 (average), and of 100 km is 0.09 (average). If we conservatively assume that the daily range of a fully charged vehicle is 75 km, then it follows from Figure 7 that most customers will be allocated a vehicle within 3 days for (M,N) = (3500, 20000); namely if $\frac{M}{N} = 0.17$. If we assume that the daily range is 100 km, it follows from Figure 6 that most customers will be allocated a vehicle within 3 days for (M,N) = (2000, 20000); namely if $\frac{M}{N} = 0.1$. From the above figures the additional cost of these vehicles is 11.4% and 6.7% of the cost of the N Nissan Leafs, respectively.

Finally, to place these figures into context, consider Table III. As can be seen, the cost of the car sharing scheme is considerably less than the (typical) level of subsidy afforded to electric vehicles in major western countries.

Country	Subsidy	Cost (Nissan Leaf)	Percentage
Ireland	€5000	€35000	14%
Belgium	€9190	€37000	25%
France	€5000	€37000	13.5%
Portugal	€5000	€35000	14%
United Kingdom	£5000	£30000	17%
United States	\$7500	\$32000	23%

TABLE III Subsidies to EV purchase (direct and indirect) and cost of Nissan Leaf [24] [25].

V. CONCLUSIONS AND FUTURE WORK

In this paper, a solution to the consumer range anxiety problem was posed using a car sharing idea. Car sharing schemes are at an advanced state of development, see for example Car2Go and DriveNow, and similar ideas could be employed here to deploy the proposed system. The cost of this scheme was shown to be low when compared with current levels of subsidies to electric vehicle manufacturers. This cost could be reduced further by: (i) exploiting car sharing on a per-day basis (further multiplexing); (ii) using low-cost vehicles (as opposed to a premium marque); (iii) selecting locations for the shared vehicles; and (iv) introducing a pricing model to regulate demand on weekends/holidays. Also, the pricing model given in Sections III and IV is primitive and represents a worst-case scenario, and more advanced modelling is required. These issues will be the subject of future work.

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