



# Hamilton Institute

Counting & Sampling Contingency Tables

**Dr. Mary Cryan**

Lab for Foundations of Computer Science  
School of Informatics  
University of Edinburgh

Wednesday, April 22nd, 2009

## **Abstract:**

Suppose we are given two lists  $r$  and  $c$  of positive integers, where  $r=(r[1], \dots, r[m])$  represents a list of prescribed 'row sums' and  $c=(c[1], \dots, c[n])$  is a list of prescribed 'column sums'. We require that  $(r[1] + \dots + r[m]) = (c[1] + \dots + c[n])$ . In this setting, we say that a  $m$ -by- $n$  matrix  $X$  of non-negative integers is a Contingency Table (for these given row/column values) if  $X$  simultaneously satisfies all of the given row and column sums. The problem of determining whether at least one contingency table exists can be solved in polynomial-time (in fact, this question is fairly trivial).

In my talk, we are interested in the more-difficult problem of randomly sampling a table uniformly at random, from the entire set of contingency tables. This problem has some applications in practical statistics which I will mention. We study a very natural Markov chain on the set of contingency tables called the 2-by-2 heat bath: one step of this chain operates by selecting 2 rows and 2 columns uniformly at random, computing the induced row sums and column sums on this 2-by-2 window, then replacing the window with a table chosen randomly from all 2-by-2 tables with the induced row and column sums. This Markov chain converges to the uniform distribution on contingency tables - our goal is to show that it approaches uniformity within polynomial-time. We are able to achieve this result for the case when the number of rows  $m$  is some fixed constant. Our proof is by application of the canonical paths method of Jerrum and Sinclair.

(Joint work with Martin Dyer, Leslie Goldberg, Mark Jerrum and Russell Martin)

**Venue:** Seminar Room, Hamilton Institute, Rye Hall,  
NUI Maynooth

**Time:** 2.00 - 3.00pm (followed by tea/coffee)

Travel directions are available at [www.hamilton.ie](http://www.hamilton.ie)