Nonregular Feedback Linearization for a Class of Second-Order Systems with Application to Flexible Joint Robots

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Abstract: In this paper, a class of second-order nonlinear cascade control systems is considered. Under certain structural assumptions, it is proven that these systems are exactly linearizable via nonregular static state feedbacks and state diffeomorphisms. Linearizing input transformations and the corresponding state diffeomorphisms are presented. Finally, nonregular static feedback linearization is applied to a class of flexible joint robots, and the controller constructed is globally asymptotically stabilizing.

Key Words: Second-order nonlinear systems, feedback linearization, nonregular state feedback, flexible joints, global stabilization.

1 Introduction

During the last decades, nonlinear systems and control theory have witnessed a tremendous development. As one of the most active research areas, feedback linearization is a powerful tool for control and synthesis of nonlinear systems, and has been widely applied to many engineering systems, for example, electrical drivers [1, 23], rigid and flexible joint robots [13, 20], spacecrafts [11, 18], to list a few.

Feedback linearization involves transforming a nonlinear system into a controllable linear one by using state feedback and coordinate transformations. This problem has been studied using increasingly more general feedback transformations. Regular static state feedback linearization was solved in [2] for single-input systems and in [9] for multi-input systems. Regular dynamic state feedback linearization was firstly proposed in [4] and then developed in [7] and the references therein. Recently, nonregular static/dynamic state feedback linearization was introduced and addressed in [22].

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Most existing results on feedback linearization are based on the (first-order) state space representation of the plant. However, many practical engineering models, such as mechanical systems, are derived by Euler-Lagrange equations or Newton-Euler formulation which usually lead to second or higher order representation. For these high order systems, the transformation to a state space format, though straightforward, may obscure some relevant model structural properties and lead to complicated expressions [14]. To this end, there is a demand to address feedback design problems based directly on the original high order models rather than on the transformed state-space representation.

In this paper, we will present a new criterion for nonregular static state feedback linearization of a class of second-order systems. Under some structural assumptions, it is shown that these systems are nonregular static state feedback linearizable, and a linearizing state feedback as well as the corresponding coordinate transformations can be obtained. As an application, globally asymptotically stabilizing controllers are presented for a class of flexible joint robots.

This paper is organized as follows. Section 2 presents a linearizability criterion for a class of second-order systems. Its application to flexible joint robots is addressed in Section 3. The last section contains some concluding remarks.

2 Main Result

Definition 1 *[22] An affine nonlinear control system*

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x)u_i = f(x) + G(x)u, \quad x \in \Re^n \tag{1}$$

with $f(0) = 0$ and rank$G(0) = m$ is said to be (locally) nonregular static state feedback linearizable (at the origin), if it can be transformed into a controllable linear system

$$\dot{z} = Az + Bv$$

via a locally defined state diffeomorphism

$$z = \phi(x), \quad \phi(0) = 0 \tag{2}$$

and a locally defined nonregular static state feedback

$$u = \alpha(x) + \beta(x)v, \quad \alpha(0) = 0,\ \beta(x) \in \Re^{m\times l},\ v \in \Re^{l},\ l \leq m \tag{3}$$

In nonregular static feedback (3), the gain matrix $\beta(x)$ is not necessarily square, and even if it is square, it is not necessarily nonsingular at the origin. In other words, "nonregular" means "not necessarily regular". It is readily seen that a regular static state feedback is also a nonregular static state feedback. Accordingly, the concept of nonregular state feedback linearization is a generalization of its regular counterpart.

For nonregular static state feedback linearization, some necessary conditions and sufficient conditions were presented in [22]. It is also shown that regular dynamic state feedback linearizability does not imply nonregular static state feedback linearizability, but the reverse does not necessarily hold. In the sequel, a new linearizability criterion for a class of second-order nonlinear cascade systems will be presented.

Consider a second-order nonlinear system given by

$$\begin{aligned} \ddot{x} &= f(x,\dot{x}) + p(x,\dot{x})y + g(x,\dot{x})u \\ \ddot{y} &= u \end{aligned} \tag{4}$$

where $x, \dot{x}, y, \dot{y} \in \Re^n$ are the states, $u \in \Re^n$ are the inputs, $f(x,\dot{x})$ is a smooth vector field, $p(x,\dot{x})$ and $g(x,\dot{x})$ are $n \times n$ matrices of real-valued functions.

Let us study the linearizability of system (4) around an equilibrium point. Without loss of generality, assume the origin is an equilibrium point of the unforced system, that is $f(0,0) = 0$. We made the following two assumptions on the system structures:

Assumption 1 *Matrix $p(x,\dot{x})$ has the upper triangular structure and is nonsingular at the origin, i.e.,*

$$p(x,\dot{x}) = \begin{bmatrix} p_{1,1}(x,\dot{x}) & p_{1,2}(x,\dot{x}) & \cdots & p_{1,n}(x,\dot{x}) \\ 0 & p_{2,2}(x,\dot{x}) & \cdots & p_{2,n}(x,\dot{x}) \\ & & \ddots & \\ 0 & 0 & \cdots & p_{n,n}(x,\dot{x}) \end{bmatrix}, \quad p_{i,i}(0,0) \neq 0,\ i = 1,\cdots,n \tag{5}$$

Assumption 2 *Matrix $g(x,\dot{x})$ has the strictly upper triangular structure and is of rank $n-1$ at the origin, i.e.,*

$$g(x,\dot{x}) = \begin{bmatrix} 0 & g_{1,2}(x,\dot{x}) & \cdots & g_{1,n}(x,\dot{x}) \\ 0 & 0 & \cdots & g_{2,n}(x,\dot{x}) \\ & & \ddots & \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \quad g_{i,i+1}(0,0) \neq 0,\ i = 1,\cdots,n-1 \tag{6}$$

Theorem 1 *System (4) satisfying Assumptions 1 and 2 is nonregular static state feedback linearizable.*

Proof. Denote $\xi = [x^T, \dot{x}^T, y^T, \dot{y}^T]^T$. Let us design the last $n-1$ input channels

$$u^2 = \begin{bmatrix} u_2 \\ \vdots \\ u_n \end{bmatrix} = \begin{bmatrix} \alpha_2(\xi) \\ \vdots \\ \alpha_n(\xi) \end{bmatrix} \tag{7}$$

such that states x and $\dot{x}$ can be linearized to be a single chain of integrator, i.e.,

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \vdots \\ \ddot{x}_{n-1} \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} \tag{8}$$

Bearing Assumptions 1 and 2 in mind, substituting (7) and (8) into (4), leads to

$$f_i(x,\dot{x}) + \sum_{j=i}^{n} p_{i,j}(x,\dot{x})y_j + \sum_{j=i+1}^{n} g_{i,j}(x,\dot{x})\alpha_j = x_{i+1} \quad i = 1, \cdots, n-1 \tag{9}$$

From which, we can solve for α_i backwardly as follows

$$\begin{aligned} \alpha_n &= c_n(x,\dot{x},y^n) + d_n(x,\dot{x})y_{n-1} \\ \alpha_{n-1} &= c_{n-1}(x,\dot{x},y^{n-1}) + d_{n-1}(x,\dot{x})y_{n-2} \\ &\vdots \\ \alpha_2 &= c_2(x,\dot{x},y^2) + d_2(x,\dot{x})y_1 \end{aligned}$$

where $y^i = [y_i, \cdots, y_n]^T$, $i = 1, \cdots, n$, and

$$\begin{aligned} & c_n(x,\dot{x},y^n) = g_{n-1,n}^{-1}(x,\dot{x})(x_n - f_{n-1}(x,\dot{x}) - p_{n-1,n}(x,\dot{x})y_n) \\ & d_n(x,\dot{x}) = -g_{n-1,n}^{-1}(x,\dot{x})p_{n-1,n-1}(x,\dot{x}) \\ & \vdots \\ & c_2(x,\dot{x},y^2) = g_{1,2}^{-1}(x,\dot{x})(x_2 - f_1(x,\dot{x}) - \sum_{j=2}^{n} p_{1,j}(x,\dot{x})y_j - \sum_{j=3}^{n} g_{1,j}\alpha_j) \\ & d_2(x,\dot{x}) = -g_{1,2}^{-1}(x,\dot{x})p_{1,1}(x,\dot{x}) \end{aligned}$$

Note that the real-valued functions $d_2(x,\dot{x}), \cdots, d_n(x,\dot{x})$ are non-zero at the origin.

The overall system of (4) and (7) becomes

$$\begin{bmatrix} \ddot{x}_1 \\ \vdots \\ \ddot{x}_{n-1} \\ \ddot{x}_n \\ \ddot{y}_1 \\ \ddot{y}_2 \\ \vdots \\ \ddot{y}_n \end{bmatrix} = \begin{bmatrix} x_2 \\ \vdots \\ x_n \\ f_n(x,\dot{x}) + p_{n,n}(x,\dot{x})y_n \\ 0 \\ c_2(x,\dot{x},y^2) + d_2(x,\dot{x})y_1 \\ \vdots \\ c_n(x,\dot{x},y^n) + d_n(x,\dot{x})y_{n-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u_1 \tag{10}$$

It is routine to verify that it satisfies the criterion for regular static state feedback linearizability (Cf, e.g., [8, Theorem 4.2.6]). Accordingly, system (10) is regular static state feedback linearizable. Furthermore, it is readily seen that $h(\xi) = x_1$ is a linearizing output. The corresponding linearizing coordinate and input transformations are

$$\begin{aligned} z &= [h, \frac{dh}{dt}, \cdots, \frac{d^{4n-1}h}{dt^{4n-1}}]^T \qquad (11) \\ &= [x_1, \dot{x}_1, \cdots, x_n, \dot{x}_n, \varphi_1 + \psi_1 y_n, \varphi_2 + \psi_2 \dot{y}_n, \cdots, \varphi_{2n-1} + \psi_{2n-1} y_1, \varphi_{2n} + \psi_{2n} \dot{y}_1]^T \end{aligned}$$

and

$$u_1 = \alpha_1(\xi) + \beta_1(\xi)v \quad v \in \Re \tag{12}$$

where

$$\begin{array}{lcllcl} \varphi_1 & = & f_n(x,\dot{x}), & \psi_1 & = & p_{n,n}(x,\dot{x}) \\ \varphi_{2i} & = & L_{\bar{f}}\varphi_{2i-1} + (L_{\bar{f}}\psi_{2i-1})y_{n+1-i}, & \psi_{2i} & = & \psi_{2i-1} \;\; i = 1, \cdots, n \\ \varphi_{2i+1} & = & L_{\bar{f}}\varphi_{2i} + (L_{\bar{f}}\psi_{2i})\dot{y}_{n+1-i} + \psi_{2i}c_{n+1-i}, & \psi_{2i+1} & = & \psi_{2i}d_{n+1-i} \;\; i = 1, \cdots, n-1 \\ \alpha_1 & = & -\dfrac{1}{\psi_{2n}}(L_{\bar{f}}\varphi_{2n} + (L_{\bar{f}}\psi_{2n})\dot{y}_1, & \beta_1 & = & \dfrac{1}{\psi_{2n}} \end{array}$$

with $L_{\bar{f}}\varphi$ denoting the derivative of φ along

$$\bar{f} = [\dot{x}_1, x_2, \dot{x}_2, \cdots, x_n, \dot{x}_n, f_n + p_{n,n}y_n, \dot{y}_1, 0, \dot{y}_2, c_2 + d_2 y_1, \cdots, \dot{y}_n, c_n + d_n y_{n-1}]^T$$

To sum up, under nonregular static state feedback

$$u = \begin{bmatrix} \alpha_1(\xi) \\ \alpha_2(\xi) \\ \vdots \\ \alpha_n(\xi) \end{bmatrix} + \begin{bmatrix} \beta_1(\xi) \\ 0 \\ \vdots \\ 0 \end{bmatrix} v = \alpha(\xi) + \beta(\xi)v \tag{13}$$

and state diffeomorphism (11), system (4) is transformed into

$$\dot{z} = Az + Bv \tag{14}$$

where (A, B) is the Brunovsky canonical matrix pair. By Definition 1, system (4) is nonregular static state feedback linearizable. $\triangle$

Remark 1 *Assumptions 1 and 2 of system (4) can be further relaxed to one assumption, Assumption 3 as follows.*

Assumption 3 *There exists a positive definite matrix $M(x, \dot{x})$ such that*

$$p(x, \dot{x}) = M(x, \dot{x})\bar{p}(x, \dot{x}), \quad g(x, \dot{x}) = M(x, \dot{x})\bar{g}(x, \dot{x})$$

where matrix $\bar{p}(x, \dot{x})$ has an upper triangular structure and is nonsingular at the origin, and matrix $\bar{g}(x, \dot{x})$ has a strictly upper triangular structure and is of rank $n-1$ at the origin.

Under this relaxed assumption, system (4) is still linearizable via nonregular static state feedback. This fact can be proven along the same line as in the proof of Theorem 1. In fact, $h(\xi) = x_1$ is a linearizing output and the corresponding linearizing input and coordinate transformations can be obtained accordingly.

Remark 2 *Theorem 1 can be readily extended to higher order systems*

$$\begin{aligned} x^{(k)} &= f(\bar{x}) + p(\bar{x})y + g(\bar{x})u \\ y^{(k)} &= u \end{aligned} \tag{15}$$

where $k > 2$, and $\bar{x} = [x^T, \dot{x}^T, \cdots, (x^{(k-1)})^T]^T$. Furthermore, they can be extended to systems in non-homogeneous orders

$$\begin{aligned} [x_1^{(k_1)}, \cdots, x_n^{(k_n)}]^T &= f(\bar{x}) + p(\bar{x})y + g(\bar{x})u \\ [y_1^{(l_1)}, \cdots, y_n^{(l_n)}]^T &= u \end{aligned} \tag{16}$$

where $k_i, l_i \geq 1$, $i = 1, \cdots, n$, and $\bar{x} = [x_1, \cdots, x_1^{(k_1-1)}, \cdots, x_n, \cdots, x_n^{(k_n-1)}]^T$.

3 Application to Flexible Joint Robots

Position control of flexible joint robot manipulators has been discussed by many researchers in the literature. Although numerous control and design strategies from diverse disciplines were developed based on simplified models (see, e.g., [3, 5, 6, 12, 16, 17] and references therein), only a few references studied the regulation issues based on the complete dynamic model derived in [24, 25]. Among these, a proportional-derivative (PD) controller is developed to globally asymptotically stabilize the robot to a reference position in [25]. Several variations of this controller can be found in [10], which replaced the derivative operator by a high pass filter, and in [15], which presented a globally stabilizing output feedback controller following the passivity approach.

In this paper, globally stabilizing controllers are presented for a class of full-order flexible joint robots within the framework of nonregular static state feedback linearization. The controllers require full state measurements which is more restrictive than the aforementioned controllers for this class of systems.

3.1 Mathematical Model

Consider a robot with $n+1$ rigid links interconnected by n flexible revolute joints. Under the assumptions that the joint flexibility are modeled as linear torsional springs and the rotors of the actuators are modeled as uniform bodies of revolution, the robot's dynamics are given by [24, 25]

$$M(q_1)\ddot{q} + C(q,\dot{q})\dot{q} + K_E q + G(q_1) + F(q,\dot{q}) = \Gamma \tag{17}$$

where $q = [q_1, q_2]$ with q_1 and q_2 representing $n \times 1$ vectors of the positions of the links and the actuators, respectively, $M(q) = \begin{bmatrix} M_1(q_1) & M_2(q_1) \\ M_2^T(q_1) & M_3 \end{bmatrix}$ is the inertia matrix which is positive definite, $C(q,\dot{q})$ is the centripetal and Coriolis matrix, $K_E = \begin{bmatrix} K_e & -K_e \\ -K_e & K_e \end{bmatrix}$ with $K_e = \mathrm{diag}[k_1, \cdots, k_n]$ being the diagonal stiffness matrix whose entries are flexible constants of the joints, $G(q_1) = \begin{bmatrix} G_1(q_1) \\ 0 \end{bmatrix}$ is the gravity term, $F(q,\dot{q}) = \begin{bmatrix} F_1(q_1,\dot{q}_1) \\ F_2(q_2,\dot{q}_2) \end{bmatrix}$ represents the friction forces, and $\Gamma = \begin{bmatrix} 0 \\ \tau \end{bmatrix}$ with τ being the torques supplied by the motors.

Matrix $M_2(q_1)$ has the strictly upper triangular structure given by

$$M_2(q_1) = \begin{bmatrix} 0 & m_{1,2}(q_{1,1}) & m_{1,3}(q_{1,1}, q_{1,2}) & \cdots & m_{1,n}(q_{1,1}, \cdots, q_{1,n-1}) \\ 0 & 0 & m_{2,3}(q_{1,2}) & \cdots & m_{2,n}(q_{1,2}, \cdots, q_{1,n}) \\ & & & \ddots & \\ 0 & 0 & 0 & \cdots & m_{n-1,n}(q_{1,n-1}) \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \tag{18}$$

Matrix $C(q,\dot{q})$ can be decomposed as

$$C(q,\dot{q}) = \begin{bmatrix} C_1(q_1,\dot{q}_1)+C_2(q_1,\dot{q}_2) & C_3(q_1,\dot{q}_1) \\ C_4(q_1,\dot{q}_1) & 0 \end{bmatrix}$$

where

$$\begin{aligned}
C_{1,ij}(q_1,\dot{q}_1) &= \frac{1}{2}\left[\dot{q}_1^T\frac{\partial M_{1,ij}}{\partial q_1} + \left(\frac{\partial M_1^i}{\partial q_{1,j}} - \frac{\partial M_1^j}{\partial q_{1,i}}\right)\dot{q}_1\right] \\
C_{2,ij}(q_1,\dot{q}_2) &= \frac{1}{2}\left(\frac{\partial M_2^i}{\partial q_{1,j}} - \frac{\partial M_2^j}{\partial q_{1,i}}\right)\dot{q}_2 \\
C_{3,ij}(q_1,\dot{q}_1) &= \frac{1}{2}\left(\dot{q}_1^T\frac{\partial M_{2,ij}}{\partial q_1} - \frac{\partial (M_2^T)^j}{\partial q_{1,i}}\dot{q}_1\right) \\
C_{4,ij}(q_1,\dot{q}_1) &= \frac{1}{2}\left(\dot{q}_1^T\frac{\partial M_{2,ji}}{\partial q_1} - \frac{\partial (M_2^T)^i}{\partial q_{1,j}}\dot{q}_1\right)
\end{aligned}$$

with M^i denoting the ith row of matrix M.

When $M_2(q_1) = 0$, there is no inertial coupling between the links and the actuators and model (17) is reduced to the well known simplified model [19]. This simplified model is always regular static state feedback linearizable. To avoid this trivial case, we assume that $M_2(q_1) \neq 0$. Furthermore, for ease of handling, we make the following assumption

Assumption 4 *Matrix $M_2(q_1)$ is constant, and $m_{i,i+1} \neq 0$, $i = 1, \cdots, n-1$.*

The constant assumption for $M_2(q_1)$ is valid for spatial three-link elbow manipulators and for planar robots with any number of rotational joints [14].

Under Assumption 4, system (17) is reduced to

$$\begin{aligned}
&M_1(q_1)\ddot{q}_1 + M_2\ddot{q}_2 + C_1(q_1,\dot{q}_1)\dot{q}_1 + K_e(q_1-q_2) + G_1(q_1) + F_1(q_1,\dot{q}_1) = 0 \\
&M_2^T\ddot{q}_1 + M_3\ddot{q}_2 + K_e(q_2-q_1) + F_2(q_2,\dot{q}_2) = \tau
\end{aligned} \tag{19}$$

Note that system (19) is not regular static state feedback linearizable [14].

3.2 Nonregular Feedback Linearizability

Consider the control law

$$\begin{aligned}
\tau = &-M_2^T M_1^{-1}(q_1)(C_1(q_1,\dot{q}_1)\dot{q}_1 + K_e(q_1-q_2) + G_1(q_1) + F_1(q_1,\dot{q}_1)) \\
&+K_e(q_2-q_1) + F_2(q,\dot{q}) + (M_3 - M_2^T M_1^{-1}(q_1)M_2)u
\end{aligned} \tag{20}$$

Note that matrix $M_3 - M_2^T M_1^{-1}(q_1)M_2$ is nonsingular (and positive definite) from the positive definiteness of $M(q_1)$.

System (19) can be rewritten in the form of partial feedback linearization [21]

$$\begin{aligned}
\ddot{q}_1 &= -M_1^{-1}(q_1)(C_1(q_1,\dot{q}_1) + K_e q_1 + G_1(q_1) + F_1(q_1,\dot{q}_1)) + M_1^{-1}(q_1)(K_e q_2 - M_2 u) \\
\ddot{q}_2 &= u
\end{aligned} \tag{21}$$

It is clear that system (21) is of form (4) and satisfies Assumption 3. According to Remark 1, this system is nonregular static state feedback linearizable.

Following the proof of Theorem 1, we can compute a linearizing state coordinate and its corresponding input transformations as

$$z = \phi(q,\dot{q}) = [q_{1,1}, \dot{q}_{1,1}, \cdots, q_{1,n}, \dot{q}_{1,n}, \eta_1, \cdots, \eta_{2n}]^T \tag{22}$$

and

$$u = \alpha + \beta v = \begin{bmatrix} -\dfrac{L_X \eta_{2n}}{\gamma_1} \\ \Lambda^{-1}\theta \end{bmatrix} + \begin{bmatrix} \dfrac{1}{\gamma_1} \\ 0 \end{bmatrix} v \tag{23}$$

respectively, where

$$\Lambda = \begin{bmatrix} m_{1,2} & \cdots & m_{1,n} \\ & \ddots & \\ 0 & \cdots & m_{n-1,n} \end{bmatrix} \quad \theta = \begin{bmatrix} k_1 q_{2,1} - D_1\rho \\ \vdots \\ k_{n-1} q_{2,n-1} - D_{n-1}\rho \end{bmatrix}$$

$$\eta_1 = a_n + \delta_n \quad \eta_{i+1} = L_X \eta_i \quad i = 1,\cdots,2n-1, \quad \gamma_1 = \frac{\partial \eta_{2n}}{\partial \dot{q}_{2,1}}$$

with $D_i = [d_{i,1},\cdots,d_{i,n}]$ being the ith row of matrix $M_1(q_1)$, and

$$\begin{aligned}
\delta &= [\delta_1,\cdots,\delta_n]^T = -M_1(q_1)^{-1}(C_1(q_1,\dot{q}_1) + K_e q_1 + G_1(q_1) + F_1(q_1,\dot{q}_1)) \\
a_n &= \frac{1}{d_{n,n}}(k_n q_{2,n} - \sum_{i=1}^{n-1} d_{n,i}(q_{1,i+1} - \delta_i)) \\
\rho &= [q_{1,2} - \delta_1, \cdots, q_{1,n} - \delta_{n-1}, \eta_1 - \delta_n]^T \\
X &= [\dot{q}_{1,1}, q_{1,2}, \cdots, \dot{q}_{1,n}, a_n - \delta_n, \dot{q}_{2,1}, 0, \dot{q}_{2,2}, \alpha_2, \cdots, \dot{q}_{2,n}, \alpha_n]^T
\end{aligned}$$

Combining (20) and (23) gives

$$\begin{aligned}
\tau = &-M_2^T M_1^{-1}(q_1)(C_1(q_1,\dot{q}_1)\dot{q}_1 + K_e(q_1 - q_2) + G_1(q_1) + F_1(q_1,\dot{q}_1)) + K_e(q_2 - q_1) \\
&+F_2(q,\dot{q}) + (M_3 - M_2^T M_1^{-1}(q_1)M_2) \begin{bmatrix} -\dfrac{L_X \eta_{2n}}{\gamma_1} \\ \Gamma^{-1}\theta \end{bmatrix} + M_3 \begin{bmatrix} \dfrac{1}{\gamma_1} \\ 0 \end{bmatrix} v
\end{aligned} \tag{24}$$

which is the overall input transformation for system (17).

The transformed system is of Brunovsky canonical form

$$\dot{z} = [z_2, \cdots, z_{4n}, v]^T \tag{25}$$

Note that the linearizing transformations (22) and (24) are globally defined. Accordingly, system (17) is globally static state feedback linearizable.

3.3 Global Stabilization

The feedback linearizability of robot system (17) enable us to design globally asymptotically stabilizing controllers by using standard linear design technique.

Suppose $p(\lambda) = \lambda^{4n} + l_{4n}\lambda^{4n-1} + \cdots + l_2\lambda + l_1$ is a Hurwitz polynomial. Let

$$v = -Lz = -\sum_{i=1}^{4n} l_i z_i \tag{26}$$

The closed-loop system (25) and (26) is globally exponential stable.

Accordingly, the controller

$$\begin{aligned} \tau = & -M_2^T M_1^{-1}(q_1)(C_1(q_1,\dot{q}_1)\dot{q}_1 + K_e(q_1 - q_2) + G_1(q_1) + F_1(q_1,\dot{q}_1)) + K_e(q_2 - q_1) \\ & +F_2(q,\dot{q}) + (M_3 - M_2^T M_1^{-1}(q_1)M_2) \begin{bmatrix} -\dfrac{L_X \eta_{2n}}{\gamma_1} \\ \Gamma^{-1}\theta \end{bmatrix} - M_3 \begin{bmatrix} \dfrac{\sum_{i=1}^{4n} l_i \phi_i(q,\dot{q})}{\gamma_1} \\ 0 \end{bmatrix} \end{aligned} \tag{27}$$

globally asymptotically stabilizing system (17).

4 Conclusion

In this paper, a new criterion of nonregular static state feedback linearization has been presented for a class of second-order nonlinear cascade systems. Some extensions of this criterion to more general systems have also been considered. As an application, controllers have been designed to asymptotically stabilize a class of flexible joint robots.

ACKNOWLEDGEMENT

The authors would like to thank the anonymous reviewers for their constructive and insightful comments for further improving the quality of this work.

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