The cp-rank of matrices generated by Soules matrices

by

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A completely positive matrix is a matrix A which can be represented as $A = BB^T$ where B is a nonnegative matrix. The cp-rank of such A is the smallest number of columns in a nonnegative matrix B satisfying $A = BB^T$. Clearly, cp-rank$A \geq$ rank$A$, and it is known that strict inequality often occurs. A Soules matrix is an orthogonal matrix R such that RDR^T is nonnegative for any nonnegative diagonal matrix D with nonincreasing diagonal elements. We show that any nonnegative matrix thus generated by a Soules matrix is a completely positive matrix whose cp-rank is equal to the rank.