## LOWER BOUNDS ON THE MINIMAL COORDINATE-DEPENDENT GROWTH RATE IN A NONNEGATIVE MULTIPLICATIVE SYSTEM

by

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## **Abstract**

This paper concerns a nonnegative  $N \times N$  matrix Q and, more generally, a system  $\{Q^{\delta} : \delta \in D\}$  of such matrices whose rows are in product form. Denote as  $\sigma(Q)$  the spectral radius of Q, and call a nonempty subset J of  $\{1, ..., N\}$  closed under Q if  $Q_{ij} = 0$  for  $i \in J$  and  $j \notin J$ . Linear systems that characterize upper bounds on  $\sigma(Q)$  and  $\max_{\delta} \sigma(Q^{\delta})$  are classic. This paper presents linear systems that characterize lower bounds on  $\min_{\delta} \min_{J} \sigma[(Q^{\delta})_{JJ}]$  where  $\delta$  ranges over D and J ranges over sets that are closed under  $Q^{\delta}$ . These results are used to characterize lower bounds on the minimal coordinate-dependent growth rate in the corresponding multiplicative systems.