Reduced Rank Nonnegative Matrix Factorization for Symmetric Nonnegative Matrices

Michael Neuamnn (Joint work with M. Catral, L. Han, and R. J. Plemmons)

Let $V \in \mathbb{R}^{m,n}$ be a nonnegative matrix. The *nonnegative matrix factorization* (NNMF) problem consists of finding nonnegative matrix factors $W \in \mathbb{R}^{m,r}$ and $H \in R^{r,n}$ such that $V \approx WH$. Lee and Seung proposed two algorithms, one of which finds nonnegative W and H such that $\|V - WH\|_F$ is minimized. After examining the case in which $r = 1$, we consider the case in which $m = n$ and V is symmetric. We focus on questions concerning when the best approximate factorization results in the product WH **being symmetric** and on cases in which the best approximation **cannot be a symmetric matrix**. We show that the class of positive semidefinite symmetric nonnegative matrices V generated via a Soules basis, admit for every $1 \leq r \leq \text{rank}(V)$, a nonnegative factorization WH which coincides with the best approximation in the Frobenius norm to V in $\mathbb{R}^{n,n}$ of rank not exceeding r.

We will also suggest applications of our findings to the *cp–rank* problem for completely positive matrices.