Infinite product of stochastic matrices and asymptotics of TCP congestion models

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Abstract

In the talk I'll consider infinite products of matrices belonging to sequences $\{Q_k\}, k \geq 0$, of row stochastic matrices. I'll present conditions for weak ergodicity of products taken in an arbitrary order of multiplication, and strong ergodicity (i.e. convergence) of the backward products $M_N = Q_N \cdots Q_1 Q_0$. Such a condition is, e.g., that the ω-limit set of $\{Q_k\}, k \geq 0$, contains a scrambling matrix. These results will be shown to be a consequence of a contraction property of certain stochastic matrices when restricted to the simplex of stochastic vectors in $\mathbb{R}^n$.

These results will be applied to the study of the congestion events of communication networks, which operate in a Transmition Control Protocol (TCP) algorithm. It will be described how arbitrary close approximations of the asymptotic equilibrium distribution of the congestion window can be computed using the above results. A second application is to the problem of existence of optimal policies of controlled Markov processes.