Subdominant eigenvalues and girth for stochastic matrices

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Abstract

Suppose that T is an $n \times n$ stochastic matrix, and that its eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$ have been labelled so that $\lambda_1 = 1$, and the moduli are in nonincreasing order. Then λ_2 is known as a subdominant eigenvalue for T, and the modulus of λ_2 governs the rate of convergence of the iterates of the Markov chain associated with T. In this talk, we explore the relationship between the modulus of the subdominant eigenvalue(s) for T, and the girth g (i.e. the length of the shortest cycle) of the directed graph associated with T. We give lower bounds on $|\lambda_2|$ in terms of g which show in particular that if g is large relative to n, then necessarily $|\lambda_2|$ is close to 1.