Nonnegativity and Stability

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A complex square matrix A is said to be stable if the spectrum of A lies in the open left or right half-plane. This, as well as other related types of matrix stability, play an important role in various applications. As such, matrix stability has been intensively investigated in the past two centuries. A plausible way for finding necessary and/or sufficient conditions for matrix stability is to examine classes of matrices that are known to be stable, and to identify common properties of these classes. Indeed, some well known classes of stable matrices share properties associated with nonnegativity, such as positivity of the principal minors (P-matrices) and weak sign symmetry. It was conjectured by Carlson that the combination P-matrix + weak sign symmetry implies stability. This conjecture was recently disproved by Holtz. However, if we replace the weak sign symmetry by the stronger sign symmetry property, then it was shown already by Carlson that P-matrix + sign symmetry implies stability. The talk will review various results that relate positivity of the principal minors, weak sign symmetry, sign symmetry and stability.