

Hall-Edmonds-Rado theorem for hyperbolic polynomials

Leonid Gurvits*

February 18, 2004

The Hall's theorem on the existence of perfect matchings is an important part of the theory of nonnegative matrices . Its extension ,the Edmonds-Rado theorem on the rank of intersection of two (geometric) matroids, belongs to the theory of positive operators [4] . I will present two generalizations of the Edmonds-Rado theorem for hyperbolic polynomials , they both are formulated in terms of the Newton polytopes and mixed hyperbolic forms . In terms of the Hall's theorem the corresponding hyperbolic polynomial is $x_1x_2\dots x_n$ and the corresponding mixed hyperbolic form is the permanent ; in terms of the Edmonds-Rado theorem the corresponding hyperbolic polynomial is $Det(X)$, where X is a hermitian matrix and the corresponding mixed hyperbolic form is the mixed discriminant .

I will also present an algorithm , hyperbolic Sinkhorn's iterative scaling , which among other things checks if the generalized Hall-Edmonds-Rado conditions hold . The complexity of this deterministic algorithm depends on one conjecture from [3] .Finally , I will describe "hyperbolic" analogues of the van der Waerden conjecture .

Time permit , I will discuss other "hyperbolic" topics such as spectral inequalities and spectral order in certain families of univariate hyperbolic polynomials .

References

- [1] L.Gurvits and A. Samorodnitsky, A deterministic algorithm approximating the mixed discriminant and mixed volume , and a combinatorial corollary, *Discrete Comput. Geom.* 27 : 531-550, 2002 .
- [2] R. B. Bapat, Mixed discriminants of positive semidefinite matrices, *Linear Algebra and its Applications* 126, 107-124, 1989.
- [3] L.Gurvits , Combinatorics hidden in hyperbolic polynomials and related topics , math.CO/0402088 , 2004 .
- [4] L. Gurvits, Classical deterministic complexity of the Edmonds problem and quantum entanglement, *Proceedings of the 35th Annual ACM Symposium on the Theory of Computing (STOC-2003)* ,San Diego ,2003.
- [5] Julius Borcea and Boris Shapiro, Hyperbolic polynomials and spectral order, preprint from Shapiro's web page.

*Los Alamos National Laboratory Email: gurvits@lanl.gov.