Nonmonic Matrix Polynomials with Nonnegative Coefficients

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Abstract

We consider matrix polynomials Q with

$$Q(z) = zI - (z^2A_2 + zA_1 + A_0) = zI - S(z)$$

for $z \in \mathbb{C}$, where A_0, A_1, A_2 are nonnegative square matrices and I is the identity matrix. The function

$$\varrho_S : \mathbb{R}_+ \to \mathbb{R}_+ \text{ with } \varrho_S(r) = \text{ spectral radius of } S(r)$$

is continuous, nondecreasing and log-convex.

Let $S(r)$ be irreducible for one (and then all) positive r. Then

1. ϱ_S is analytic on $]0, \infty[$.
2. $\varrho_S(r) = r$ for all $r \geq 0$ or there exist at most two positive r with $\varrho_S(r) = r$.
3. $\varrho_S(r) = r$ for all $r \geq 0$ if and only if
$$\sigma(Q) = \{z \in \mathbb{C} : Q(z) \text{is singular}\} = \mathbb{C}.$$
4. If $\varrho_S(r) = r$ for exactly two positive r_1 and r_2 with $r_1 < r_2$, then
$$\sigma(Q) \cap \{z \in \mathbb{C} : r_1 < |z| < r_2\} = \emptyset,$$
and Q has the same finite number of spectral values on the circles with radii r_1 and r_2, respectively; this number can be characterized by the length of cycles in an appropriate infinite graph.
5. A nonnegative right root of Q is irreducible or has zero columns.
6. Q has a nonnegative nonzero right root if and only if $\varrho_S(r) = r$ for some $r > 0$.
7. $\varrho_S(r) < r$ for some positive r implies
$$Q(z) = (B - zA_2)(zI - W)$$
for some nonnegative W and some invertible B with nonnegative inverse. W is a spectral root of Q with spetral radius less then r.

This is a joint work with B. Nagy, Budapest, Hungary.