

A talk by Michael Neumann on:

Transition Matrices for Well-Conditioned Markov Chains

This is joint work with Stephen Kirkland of the University of Regina in Canada and Jianhong Xu of the University of Southern Illinois in Carbondale

Abstract

Let $T \in \mathbb{R}^{n \times n}$ be an irreducible stochastic matrix with stationary distribution vector π . Set $A = I - T$, and define the quantity $\kappa_3(T) \equiv \frac{1}{2} \max_{j=1, \dots, n} \pi_j \|A_j^{-1}\|_\infty$, where A_j , $j = 1, \dots, n$, are the $(n-1) \times (n-1)$ principal submatrices of A obtained by deleting the j -th row and column of A . Results of Cho and Meyer, and of Kirkland show that κ_3 provides a sensitive measure of the conditioning of π under perturbation of T . Moreover, it is known that $\kappa_3(T) \geq \frac{n-1}{2n}$.

In this paper, we investigate the class of irreducible stochastic matrices T of order n such that $\kappa_3(T) = \frac{n-1}{2n}$, for such matrices correspond to Markov chains with desirable conditioning properties. We identify some restrictions on the zero-nonzero patterns of such matrices, and construct several infinite classes of matrices for which κ_3 is as small as possible.