Bounds on exponents of K-primitive matrices

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Let K be a proper cone in $\mathbb{R}^n$. Let $\pi(K)$ denote the set of all K-nonnegative matrices, that is, all $A \in \mathbb{R}^{n\times n}$ which map K into K. We say that $A \in \pi(K)$ is *K-positive* provided that $Ax \in intK$ for every $0 \neq x \in K$, and it is *K-primitive* provided that there exists a positive integer ℓ such that A^ℓ is K-positive. The *exponent* of a K-primitive matrix $A, \gamma(A)$, is the smallest positive integer ℓ such that A^ℓ is K-positive.

When $K = \mathbb{R}^n_+$, the nonnegative orthant, the notions of a K-nonnegative, K-positive and K-primitive matrix reduce to the well known notions of a nonnegative (elementwise), positive and primitive matrix.

Suppose that K has m extreme rays. We prove that if A is K-primitive then $\gamma(A) \leq (m-1)(n-1)+1$. This upper bound reduces to Wielandt's well known bound in case $m = n$. We discuss the case where the upper bound for $\gamma(A)$ is attained, and describe a gap in the set of possible values for $\gamma(A)$. An important tool in the discussion is a directed graph that can be associated with each $A \in \pi(K)$, and whose vertices are the extreme rays of K.

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