Comparative Stability and Interpolation

Izchak Lewkowicz, ECE department, Ben-Gurion University, Israel.

Let $\mathbf{H}(A)$ describe the set of all quadratic Lyapunov functions associated with a given stable linear time-invariant state-space dynamical system $\frac{dx}{dt} = Ax$. Specifically,

$$\mathbf{H}(A) := \{H = H^* \ : \ HA + A^*H \in \mathbf{P} \ \},$$

where $\mathbf{P}$ is the set of all positive definite matrices. Thus, for a pair of matrices A and B, the relation $\mathbf{H}(A) \subset \mathbf{H}(B)$ may be interpreted as having the system $\frac{dx}{dt} = Bx$ "more stable" than $\frac{dx}{dt} = Ax$.

More generally, if the matrix A has no imaginary eigenvalues, the set $\mathbf{H}(A)$ is not empty, and all matrices in it, together with A, share the same inertia.

In a series of papers in the mid 1970's R. Loewy characterized the case where two matrices A and B share the same set of Lyapunov factors: Using a different terminology, he showed that $\mathbf{H}(A) = \mathbf{H}(B)$, if and only if $\mathcal{C}_v(A) = \mathcal{C}_v(B)$, namely A and B generate the same vertical convex invertible cone (in the real case, $\mathbf{H}_r(A) = \mathbf{H}_r(B)$ is equivalent to $\mathcal{C}(A) = \mathcal{C}(B)$, i.e. A and B generate the same convex invertible cone).

Here we extend Loewy's results to $\mathbf{H}(A) \subseteq \mathbf{H}(B)$ and to the real analogue $\mathbf{H}_r(A) \subseteq \mathbf{H}_r(B)$ and show that they are equivalent to $\mathcal{C}_v(A) \subseteq \mathcal{C}_v(B)$ (or in the real case $\mathcal{C}(A) \subseteq \mathcal{C}(B)$). Furthermore, it turns out to that each of these characterizations may be casted in the framework of the classical Nevanlinna-Pick interpolation problem (complex and real, respectively).

A joint work with Nir Cohen, Mathematics department, University of Campinas, Brazil.