EE304 Probability and Statistics - Tutorial 2 Solutions

Question 1

(i) A country has 7 major cities. Each pair of these cities is connected by a direct rail link. How many rail links are needed?

Solution: The number of rail links is equal to the total number of pairs of cities. This is $\binom{7}{2} = \frac{7(6)}{2(1)} = 21$.

(ii) How many distinct arrangements of the word "allegorical" are possible?

Solution: The total number of letters is 11. However, there are 3 'l's and 2 'a's. Therefore the total number of distinct arrangements is $\frac{11!}{3!2!}$.

(iii) A password for a computer system must consist of 3 letters (all lower case) followed by 3 digits. Letters and digits can be repeated. How many distinct passwords are possible?

Solution: As letters can be repeated and the order of selection matters, we can choose the first 3 letters in 26^3 ways. Similarly the three digits can be selected in 10^3 ways. The total number of passwords is then $(26^3)(10^3)$.

(iv) A password for a computer system must consist of 6 distinct characters, 3 of which must be letters and 3 of which must be digits. How many distinct passwords are possible?

Solution: We can choose the 3 letters in $\binom{26}{3}$ ways and the 3 digits in $\binom{10}{3}$ ways. For each selection of 3 distinct letters and 3 distinct digits, there are 6! distinct passwords corresponding to different rearrangements of the characters. Thus the answer is $6!\binom{26}{3}\binom{10}{3}$.

(v) A password consists of 6 distinct characters, at least one of which must be a digit and at least 1 of which must come from the special characters #, *, &. How many passwords are possible?

Solution:

There are a total of 39=26+10+3 characters to choose from. The total number of different arrangements of these characters taken 6 at a time is $6!\binom{39}{6}$. We next calculate the total number of arrangements that do NOT contain at least 1 special character and at least 1 digit.

This will be given by

$$6!(\binom{29}{6} + \binom{36}{6} - \binom{26}{6}).$$

Therefore the answer is

$$6!(\binom{39}{6} - \binom{29}{6} - \binom{36}{6} + \binom{26}{6}).$$

There is also a much longer way of doing this by counting separately all of the different ways in which we can get at least 1 special character and at least 1 digit. It is worth checking that this will give the same answer as we have here. (It will.)

Question 2

3 Maths books, 4 History books and 2 Computing books are to be arranged side by side on a shelf. If the arrangement is done randomly so that every arrangement is equally likely calculate the probability that:
(i) the 3 maths books are all next to each other;

Solution:
The total number of different arrangements is 9!. If the three maths books are next to each other in a specified order we would have a total of 7! arrangements (treating the maths books as a single book). However for each of these, we can permute the maths books in 3! ways giving a total of 3!7!. The desired probability is

$$\frac{3!7!}{9!}.$$

(ii) the 2 books at the ends of the arrangement are computing books;
Solution:
There are 2 possible arrangements of the computing books at the ends. For each of these, there are a total of 7! arrangements of the books in the centre. Thus, the desired probability is

$$\frac{2!7!}{9!}.$$

(iii) the two books at the ends of the arrangement are History books.
Solution:
There are $4(3) = 12$ possible arrangements of the two "end" books. For each

of these, there are 7! arrangements of the books in the centre. Thus the desired probability is

$$\frac{12(7!)}{9!}.$$

Question 3

A simple communications system sends the two symbols 0, 1 over a noisy channel. The probability of a bit being received incorrectly is 0.05. If 55% of all the bits transmitted are 0, calculate:

(i) the probability that a 1 is recieved;

Solution:

Let $1R, 1S$ denote the events that a 1 is received and sent respectively. Similarly, let $0R, 0S$ be the events that a 0 is received and sent. Then from the law of total probability

$$\begin{aligned} P(1R) &= P(1R|1S)P(1S) + P(1R|0S)P(0S) \\ &= (0.95)(0.45) + (0.05)(0.55) = 0.455. \end{aligned}$$

(ii) the probability that a 0 is received.

Solution:

This can be computed in the same way as above but also as every bit received is either 0 or 1, we have

$$P(0R) = 1 - P(1R) = 0.545.$$

If the probability of a transmitted 1 being received as a 0 is 0.05 but the probability of a 0 being received as a 1 is 0.08 and a 1 is received, what is the probability that a 1 was actually transmitted?

Solution:

We want $P(1S|1R)$. By Bayes law this is

$$\begin{aligned} P(1S|1R) &= \frac{P(1R|1S)P(1S)}{P(1R|1S)P(1S) + P(1R|0S)P(0S)} \\ &= \frac{(0.95)(0.45)}{(0.95)(0.45) + (0.92)(0.55)} \\ &= 0.45795. \end{aligned}$$