EE304 Probability and Statistics - Tutorial 1 Solutions

Question 1

A fair coin is tossed three times. What is the probability of getting:

(i) exactly two heads?
Solution:
There are a total of $2 \times 2 \times 2 = 8$ possible outcomes. The outcomes corresponding to exactly 2 heads are HHT, HTH, THH. So the desired probability is $\frac{3}{8}$.

(ii) exactly 1 tail?
Solution:
This is again equal to $\frac{3}{8}$ as getting exactly 1 tail means that we have exactly two heads!

(iii) at least 1 tail?
Solution:
The only way we will not get at least 1 tail is if we get 3 heads. Thus the desired probability is

$$1 - \frac{1}{8} = \frac{7}{8}.$$

Question 2

A 4 sided fair die is thrown twice. What is the probability that the sum of the two outcomes is:

(i) equal to 6?
Solution: The total number of possible outcomes is $4 \times 4 = 16$. The outcomes giving a sum equal to 6 are $(2, 4), (3, 3), (4, 2)$. So the desired probability is $\frac{3}{16}$.

(ii) at least 6?
Solution: The outcomes giving a sum of 7 are $(3, 4), (4, 3)$ and the only outcome giving a sum of 8 is $(4, 4)$. So the desired probability is $\frac{6}{16}$.

(iii) at most 2?
Solution: We cannot get a sum equal to 1. So the only outcome giving a sum at most 2 is $(1, 1)$. The desired probability is $\frac{1}{16}$.

Question 3
An unfair 6-sided die has the following properties. Each of the numbers 1, 3, 5 is equally likely; each of the numbers 2, 4, 6 is equally likely; an even number is twice as likely to occur as an odd number. If this die is rolled once, what is the probability that the result is:

(i) a number strictly less than 4?;
Solution: Let p be the probability of getting a 1. Then $P(3) = P(5) = P(1) = p$ and $P(2) = P(4) = P(6) = 2p$. As all the probabilities must sum to 1, we find that $9p = 1$ so $p = 1/9$. Therefore the probability that the outcome is a number strictly less than 4 is

$$P(1) + P(2) + P(3) = 1/9 + 2/9 + 1/9 = 4/9.$$

(ii) divisible by 3?.
Solution: This is

$$P(3) + P(6) = 1/9 + 2/9 = 3/9 = 1/3.$$

Question 4
E, F, G are events with $P(E) = 0.3$, $P(F) = 0.4$, $P(G) = 0.6$. Also, $P(E \cap F) = 0.1$, $P(E \cap G) = 0.2$, $P(F \cap G) = 0.2$, $P(E \cup F \cup G) = 0.9$. What is:

(i) $P(E \cup F)$?;
Solution:

$$\begin{aligned} P(E \cup F) &= P(E) + P(F) - P(E \cap F) \\ &= 0.3 + 0.4 - 0.1 = 0.6. \end{aligned}$$

(ii) $P(E \backslash F)$?;
Solution:

$$\begin{aligned} P(E \backslash F) &= P(E) - P(E \cap F) \\ &= 0.3 - 0.1 = 0.2. \end{aligned}$$

(iii) $P(E \cap F \cap G)$?.
Solution:

$$\begin{aligned} P(E \cup F \cup G) &= P(E \cup F) + P(G) - P((E \cup F) \cap G) \\ \Rightarrow 0.9 &= 0.6 + 0.6 - P((E \cup F) \cap G). \end{aligned}$$

So
$$P((E \cup F) \cap G) = P((E \cap G) \cup (F \cap G)) = 0.3.$$
But
$$\begin{aligned} P((E \cap G) \cup (F \cap G)) &= P(E \cap G) + P(F \cap G) - P(E \cap F \cap G) \\ \Rightarrow 0.3 &= 0.2 + 0.2 - P(E \cap F \cap G). \end{aligned}$$
So $P(E \cap F \cap G) = 0.1$.