

## EE304 - Assignment 2

Due by 12 noon on Monday, November 15th

*Late Submission will be Penalised*

### Question 1

A fair 8-sided die is rolled twice. The random variable  $X$  is the higher of the two numbers that come up.

- (i) What is the range of the random variable  $X$ ?
- (ii) Write down in a table the probability mass function of  $X$ .
- (iii) Write down the cumulative distribution function for  $X$ .
- (iv) Calculate the mean and variance of  $X$ .

### Question 2

A continuous random variable  $X$  has probability density function given by

$$f(x) = \begin{cases} 0 & x < 0 \\ cx^3 & 0 \leq x < 5 \\ 0 & x > 5 \end{cases}$$

- (i) What is the value of the constant  $c$ ?
- (ii) What is the cumulative distribution function of  $X$ ?
- (iii) Calculate  $P(1 < X < 2)$ .
- (iv) Calculate the expected value and the variance of  $X$ .
- (v) What is  $E[X^2 + 2X]$ ?

### Question 3

It is known that 5% of all laptops from a certain manufacturer have a certain defect. A random sample of 20 laptops from this manufacturer is selected.

- (i) What is the probability that no laptops in the sample have the defect?
- (ii) What is the probability that exactly 2 laptops in the sample have the defect?

- (iii) What is the probability that at most 2 laptops in the sample have the defect?
- (iv) Let  $X$  denote the number of defective laptops in a sample. What is the expected value of  $X$ ,  $E[X]$ ?

Laptops from this manufacturer are sold in batches of 12 and a batch is deemed to be unsatisfactory if it contains 2 or more laptops with the defect. If 5 batches are selected at random, what is the probability that at least 2 of them are deemed unsatisfactory?

**Question 4**

The number of cars passing an automatic toll point on a motorway is modelled as a Poisson process with a mean of 3 cars passing per minute.

- (i) What is the probability of no cars passing the toll in an interval of 30 seconds?
- (ii) What is the probability of 3 or more cars passing the toll in an interval of 45 seconds?
- (iii) Determine the length of a time interval (in minutes) such that the probability of no car passing the toll point in this interval is 0.4.
- (iv) What is the probability of an even number of cars passing the toll in an interval of 1 minute?

**Question 5**

The daily demand for petrol at a filling station is normally distributed with a mean of 10,000 litres per day and a standard deviation of 1,500 litres per day. The total storage capacity of the filling station is 13,000 litres. The station is full to capacity at opening time each morning.

- (i) What is the probability that the filling station will be unable to meet demand on a given day?
- (ii) If we assume that demand levels on different days are independent, what is the probability that the station will fail to meet demand on exactly two days in a 5 day period?
- (iii) Determine the storage capacity needed to ensure that the probability that the station will be unable to meet daily demand is 0.025.