EE304 - Assignment 1 Solutions

Question 1

E, F and G are events with $P(E) = 0.65$, $P(F) = 0.4$, $P(G) = 0.6$, $P(E \cup F \cup G) = 1$. Further, $P(E \cap F) = 0.3$, $P(E \cap G) = 0.35$, $P(F \cap G) = 0.2$.
Calculate:

(i) $P(E \cup F)$;
Solution:

$$\begin{aligned} P(E \cup F) &= P(E) + P(F) - P(E \cap F) \\ &= 0.65 + 0.4 - 0.3 = 0.75. \end{aligned}$$

(ii) $P(F \cup G)$;
Solution:

$$\begin{aligned} P(F \cup G) &= P(F) + P(G) - P(F \cap G) \\ &= 0.4 + 0.6 - 0.2 = 0.8. \end{aligned}$$

(iii) $P(E \cap F \cap G)$;
Solution:

$$P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E \cap F) - P(E \cap G) - P(F \cap G) + P(E \cap F \cap G)$$

which implies that

$$\begin{aligned} P(E \cap F \cap G) &= 1 - 0.65 - 0.4 - 0.6 + 0.3 + 0.35 + 0.2 \\ &= 0.2 \end{aligned}$$

(iv) the conditional probability $P(E|F)$;
Solution:

$$\begin{aligned} P(E|F) &= \frac{P(E \cap F)}{P(F)} \\ &= \frac{0.3}{0.4} = 0.75. \end{aligned}$$

(v) the conditional probability $P(E|F \cup G)$;
Solution:

$$\begin{aligned} P(E|F \cup G) &= \frac{P(E \cap (F \cup G))}{P(F \cup G)} \\ &= \frac{0.3 + 0.35 - 0.2}{0.8} \\ &= \frac{0.45}{0.8} = 0.5625. \end{aligned}$$

(vi) the probability that *exactly one* of the events E, F, G occurs.
Solution:
The probability that E and F occur but not G is

$$P(E \cap F) - P(E \cap F \cap G) = 0.1.$$

Similarly, the probability that E and G occur but not F is

$$0.35 - 0.2 = 0.15$$

and the probability of both F and G occurring but not E is 0. Therefore the probability of exactly two of the events occurring is 0.25.

The probability of all three events occurring is 0.2. As $P(E \cup F \cup G) = 1$, it follows that the probability of exactly one event occurring is

$$1 - 0.25 - 0.2 = 0.55.$$

Question 2

A 6 sided die is unfair and the probability of each number i is proportional to i. (This means that the probability $P(X = i) = k \times i$ for some fixed number k.) If this die is rolled twice, what is the probability of:

(i) getting a 1 on both rolls;
Solution:
We must first work out the value of k. We know that

$$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$$

and hence

$$k(1 + 2 + 3 + 4 + 5 + 6) = 1$$

which implies that $k = 1/21$. Thus the probability of getting 1 on both rolls is

$$(1/21)(1/21) = 1/441.$$

(ii) the sum of the two outcomes being an even number;
Solution:
We will get an even number if the outcomes are

$$(1,1),(1,3),(1,5),(2,2),(2,4),(2,6),(3,1),(3,3),(3,5),$$
$$(4,2),(4,4),(4,6)(5,1),(5,3),(5,5),(6,2),(6,4),(6,6).$$

To calculate the probability that the sum is an even number, we simply work out the probability of each of the above outcomes and add all of these together. For instance, the probability of $(1,5)$ is $(1/21)(5/21) = 5/441$ while the probability of $(4,2)$ is $(4/21)(2/21) = 8/441$.

When we work out all the probabilities and sum them up, we find the probability of getting an even sum is

$$\frac{225}{441} = \frac{25}{49} = 0.5102.$$

(iii) the second outcome being 5 given that the sum of the two outcomes is even;

Solution:

Let E be the event that the sum is even and F denote the event that the second outcome is 5. Then we want $P(F|E) = \frac{P(E \cap F)}{P(E)}$. The outcomes in $E \cap F$ are $(1,5),(3,5),(5,5)$. Thus $P(E \cap F) = \frac{45}{441}$ and the desired probability is

$$P(F|E) = \frac{45}{225} = 0.2.$$

(iv) the sum of the outcomes being even given that the second outcome is 5.

Solution:

This time, we want $P(E|F)$. As before the probability $P(E \cap F) = \frac{45}{441}$. The outcomes corresponding to F are $(1,5),(2,5),(3,5),(4,5),(5,5),(6,5)$. Thus the probability $P(F) = \frac{105}{441}$ and the desired probability is

$$P(E|F) = \frac{45}{105} = \frac{3}{7}.$$

Question 3

A box of 25 mobile phones contains 10 phones equipped with WiFi but without a camera, 9 phones equipped with a camera but without WiFi and 6 phones equipped with both. If 5 phones are chosen randomly (without replacement) from this box, what is the probability that:

(i) None of the 5 phones has both WiFi and a camera?

Solution:

The total number of ways of choosing 5 phones, none of which have both WiFi and a camers, is $\binom{19}{5}$. The total number of ways of choosing 5 phones from 25 is $\binom{25}{5}$. So the answer is

$$\frac{\binom{19}{5}}{\binom{25}{5}}.$$

(ii) All 5 phones have a camera?

Solution:

15 of the 25 phones have a camera. So the total number of ways of choosing 5 phones with a camera is $\binom{15}{5}$. Thus, the answer is

$$\frac{\binom{15}{5}}{\binom{25}{5}}.$$

(iii) At least one of the phones has WiFi?
Solution:
The total number of ways of choosing 5 phones, none of which has WiFi, is $\binom{9}{5}$. Thus the probability that at least one phone has WiFi is

$$1 - \frac{\binom{9}{5}}{\binom{25}{5}}.$$

(iv) Exactly two phones have WiFi but no camera, and the other three phones have a camera?
Solution:
Number of ways of choosing 2 phones with WiFi but no camera is $\binom{10}{2}$. The number of ways of choosing 3 phones with a camera is $\binom{15}{3}$. Thus the probability required is

$$\frac{\binom{10}{2}\binom{15}{3}}{\binom{25}{5}}.$$

(v) At least 3 phones have both WiFi and a camera.
Solution:
The probability that exactly 3 have both WiFi and a camera is

$$\frac{\binom{6}{3}\binom{19}{2}}{\binom{25}{5}}.$$

The probability that exactly 4 phones have both WiFi and a camera is

$$\frac{\binom{6}{4}\binom{19}{1}}{\binom{25}{5}}.$$

The probability that exactly 5 phones have both WiFi and a camera is

$$\frac{\binom{6}{5}}{\binom{25}{5}}.$$

Thus the desired probability is

$$\frac{\binom{6}{3}\binom{19}{2}}{\binom{25}{5}} + \frac{\binom{6}{4}\binom{19}{1}}{\binom{25}{5}} + \frac{\binom{6}{5}}{\binom{25}{5}}.$$

Question 4

A company manufacturing laptops installs one of three operating systems on each laptop produced. It is known from product testing that during an hour of web browsing the probability a laptop with system 1 installed will crash is 0.15, the probability of a laptop with operating system 2 crashing is 0.08 and the probability of a laptop with system 3 crashing is 0.1. Operating systems 1 and 3 are installed on the same number of laptops while system 2 is installed on twice as many laptops as system 1 (or system 3).

(i) What is the probability that a randomly selected laptop crashes during an hour of web browsing?

Solution:

Let S_1, S_2, S_3 denote the events that the laptop has system 1, system 2, system 3 installed respectively. Let C denote the event that the laptop crashes.

$$\begin{aligned} P(C) &= P(C|S_1)P(S_1) + P(C|S_2)P(S_2) + P(C|S_3)P(S_3) \\ &= (0.15)(0.25) + (0.08)(0.5) + (0.1)(0.25) \\ &= 0.1025. \end{aligned}$$

(ii) If a laptop selected at random crashed during an hour of browsing the web, what is the probability it has operating system 2 installed?

Solution:

$$\begin{aligned} P(S_2|C) &= \frac{P(C|S_2)P(S_2)}{P(C)} \\ &= \frac{(0.08)(0.5)}{0.1025} \\ &= 0.3902. \end{aligned}$$

(iii) If a laptop selected at random does not crash during an hour of web browsing what is the probability it has operating system 1 or operating system 2 installed?

Solution:

By Bayes' Theorem:

$$\begin{aligned} P(S_1|C^c) &= \frac{P(C^c|S_1)P(S_1)}{P(C^c)} \\ &= \frac{(0.85)(0.25)}{0.8975} \\ &= 0.2368. \end{aligned}$$

$$
\begin{aligned}
P(S_2|C^c) &= \frac{P(C^c|S_2)P(S_2)}{P(C^c)} \\
&= \frac{(0.92)(0.5)}{0.8975} \\
&= 0.5125.
\end{aligned}
$$

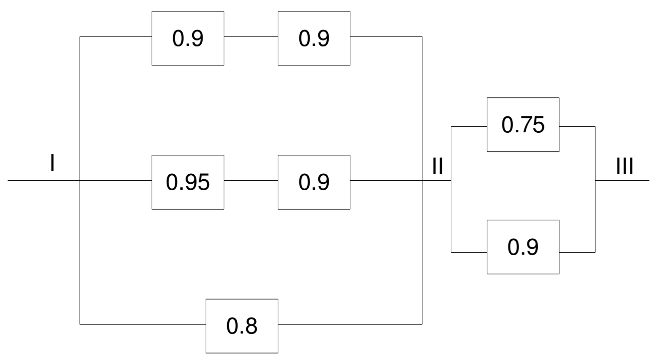
Thus the desired probability is

$$0.5125 + 0.2368 = 0.7493.$$

Question 5

The circuit in the diagram below functions provided there is a path of functional devices from left to right. The probability of each device functioning is shown and the devices operate independently of each other. What is the probability that the circuit functions?

Solution:



First we work out the probability that the section from I to II works. This section will work unless all 3 branches fail. The top branch fails with probability

$$1 - (0.9)(0.9) = 1 - 0.81 = 0.19.$$

The middle branch fails with probability

$$1 - (0.95)(0.9) = 0.145.$$

The bottom branch fails with probability

$$1 - 0.8 = 0.2.$$

Thus the probability the section from I to II fails is $(0.2)(0.19)(0.145) = 0.00551$ and the probability that it works is $1 - 0.00551 = 0.99449$.

Next we work out the probability that the section from II to III works. The probability that this section fails is

$$(0.25)(0.1) = 0.025.$$

Thus the probability that this section works is

$$1 - 0.025 = 0.975.$$

Hence the probability that the whole circuit works is

$$(0.975)(0.99449) = 0.9696.$$