July 14, 2011

Introduction to Dynamic Systems, Summer 2011

TEST

Problem 1 Obtain a state-space description of the following system.

$$\begin{aligned} \ddot{q}_1 + 2\dot{q}_2 &= -q_1^3 + q_2 + 2u \\ 2\ddot{q}_1 + \dot{q}_2 &= -2q_1^3 + 2q_2 + u \\ y &= \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \end{aligned}$$

Problem 2 What are the solutions to

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -x_2 \end{aligned}$$

with the initial states

$$x(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}.$$

at initial time 0?

Problem 3 Determine the stability properties of the following system.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= x_1 \end{aligned}$$

Problem 4 Using linearization determine (if possible) the stability properties of each of the following systems about their corresponding specified equilibrium solution q^e. If not possible, provide a reason.

(a)

$$\begin{aligned} (1+q_1^2)\ddot{q}_1 - \dot{q}_1 + \dot{q}_2 - \sin q_2 &= 0 \\ 2\ddot{q}_1 + \dot{q}_1\dot{q}_2 + \dot{q}_1 - \dot{q}_2 - 2\sin q_2 &= 0 \end{aligned}$$

and $q_1^e = q_2^e = 0$.

(b)

$$\ddot{q} + \dot{q}^3 + q^3 = 0$$

and $q^e = 0$.

Problem 5 Consider the system with input u, output y, and state variables x_1, x_2 described by

$$\begin{aligned}\dot{x}_1 &= x_2 + u \\ \dot{x}_2 &= x_1 + u \\ y &= x_1 + x_2\end{aligned}$$

(a) Is this system observable?
(b) If the system is unobservable, determine its unobservable eigenvalues.

Problem 6 Consider the system with control input variables u_1, u_2 and state variables x_1, x_2, x_3 described by

$$\begin{aligned}\dot{x}_1 &= x_2 + u_1 + u_2 \\ \dot{x}_2 &= x_3 + u_1 \\ \dot{x}_3 &= x_2 + u_1\end{aligned}$$

(a) Is this system controllable?
(b) If the system is uncontrollable, determine its uncontrollable eigenvalues.

Problem 7 Consider a discrete-time system described by

$$\begin{aligned}x_1(k+1) &= x_2(k) + u(k) \\ x_2(k+1) &= x_1(k) + 2u(k)\end{aligned}$$

Obtain a state feedback controller which always results in the state of the resulting closed loop system going to zero in a finite number of steps.

Problem 8 Consider the scalar-input scalar-output system described by

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= 4x_1 + u \\ y &= -x_1 + x_2\end{aligned}$$

Obtain an output feedback controller which stabilizes this system about the zero state.

Problem 9 Consider the input-output system described by

$$\ddot{y} + 3\dot{y} + 2y = \dot{u} - u$$

Is there a nonzero input u which does not decay to zero but, for all initial conditions on y and $\dot{y}$, results in an output which decays to zero? If the answer is yes, give an example of such an input.