Correction to "D-Stability and Delay-Independent Stability of Homogeneous Cooperative Systems"

Vahid Samadi Bokharaie, Oliver Mason and Mark Verwoerd

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***Abstract*—We correct some errors in the statements and proofs presented in Section V of the above mentioned manuscript.**

In the recent paper [1], the results of Section V are incorrect as stated. In this brief note, we first present an example to show that the statement of Theorem 5.1 in [1] is incorrect and then state a corrected version of this result under an additional technical assumption on the vector field f.

Example 1

Consider the 2-dimensional cooperative system given by

$$\dot{x} = f(x_1, x_2) = \begin{pmatrix} -\frac{x_1}{1+x_1} + x_2 \\ -\frac{x_2}{1+x_2} \end{pmatrix}.$$

Using a similar argument as employed in Example 3.19 of [2], it can be shown that the origin is a globally asymptotically stable equilibrium of this system. However, it is clear that choosing $a = (1, 2)$, there can be no vector $v \geq a$ with $f(v) \ll 0$.

The proof of Theorem 5.1 in [1] implicitly assumes that for any $a \in \mathbb{R}^n_+$, both of the sets

$$\begin{aligned} \Omega_1 &:= \{x \geq a : f_1(x) < 0\} \\ \Omega_2 &:= \{x \geq a : f_2(x) < 0\} \end{aligned} \quad (1)$$

are *non-empty*. In general, this will not be the case as is clear from the above example.

In order for the conclusion of Theorem 5.1 in [1] to hold, it is necessary to impose some additional assumption on the vector field f.

Assumption A:

(i) For any $x_1 \in \mathbb{R}_+$, there exists some K_{x_1} and $\epsilon_{x_1} > 0$ such that $\frac{\partial f_2}{\partial x_2}(x_1, t) \leq -\epsilon_{x_1}$ for all $t \geq K_{x_1}$.

(ii) For any $x_2 \in \mathbb{R}_+$, there exists some K_{x_2} and $\epsilon_{x_2} > 0$ such that $\frac{\partial f_1}{\partial x_1}(t, x_2) \leq -\epsilon_{x_2}$ for all $t \geq K_{x_2}$.

In the following result, which is a correction of Theorem 5.1 in [1], f is required to satisfy the same conditions as in Section V of [1] as well as Assumption A above.

Theorem 5.1 Assume that the system $\dot{x}(t) = f(x(t))$ has a GAS equilibrium at the origin. Then given any $a \in \mathbb{R}^2_+$, there exists $v \geq a$ with $f(v) \ll 0$.

Proof: If there exist $u \geq a$ and $w \geq a$ with $f_1(u) < 0$ and $f_2(w) < 0$ then the sets Ω_1, Ω_2 in (1) are both non-empty and the argument presented in Theorem 5.1 of [1] implies the existence of a $v \geq a$ with $f(v) \ll 0$. We now prove that, provided Assumption A is satisfied, these sets are both non-empty.

We shall show that there exists $u \geq a$ with $f_1(u) < 0$. (The proof that there is a $w \geq a$ with $f_2(w) < 0$ is identical.) By Assumption A, there exists some constants K, $\epsilon > 0$ such that

$$\frac{\partial f_1}{\partial x_1}(t, a_2) < -\epsilon$$

for all $t \geq K$. For $t \geq K$, we have

$$\begin{aligned} f_1(t, a_2) &= f_1(K, a_2) + \int_K^t \frac{\partial f_1}{\partial x_1}(s, a_2) ds \\ &\leq f_1(K, a_2) - \epsilon(t - K). \end{aligned}$$

This immediately implies that for

$$t > K + \frac{f_1(K, a_2)}{\epsilon},$$

we must have $f_1(t, a_2) < 0$. This completes the proof.

Provided the vector field f satisfies the additional Assumption A and g is non-decreasing, the other results of Section V of [1] are correct as stated.

References

[1] Bokharaie, V.S., Mason, O. and Verwoerd, M. Stability Properties of Equilibria of Classes of Cooperative Systems. *IEEE Transactions on Automatic Control*, 55(12):1996–2001, 2010.

[2] Ruffer, B. S., Kellett, C. M. and Weller, S. R. Connection between cooperative positive systems and integral input-to-state stability of large-scale systems *Automatica*, 46(6): 1019-1027, 2010.

V. S. Bokharaie is with Hamilton Institute, National University of Ireland, Maynooth, Ireland (email: vahid.bokharaie@nuim.ie)

O. Mason is with Hamilton Institute, National University of Ireland, Maynooth, Ireland (email: oliver.mason@nuim.ie)

M. Verwoerd was with Hamilton Institute, National University of Ireland, Maynooth, Ireland