Lake eutrophication: using resilience evaluation to compute sustainable policies

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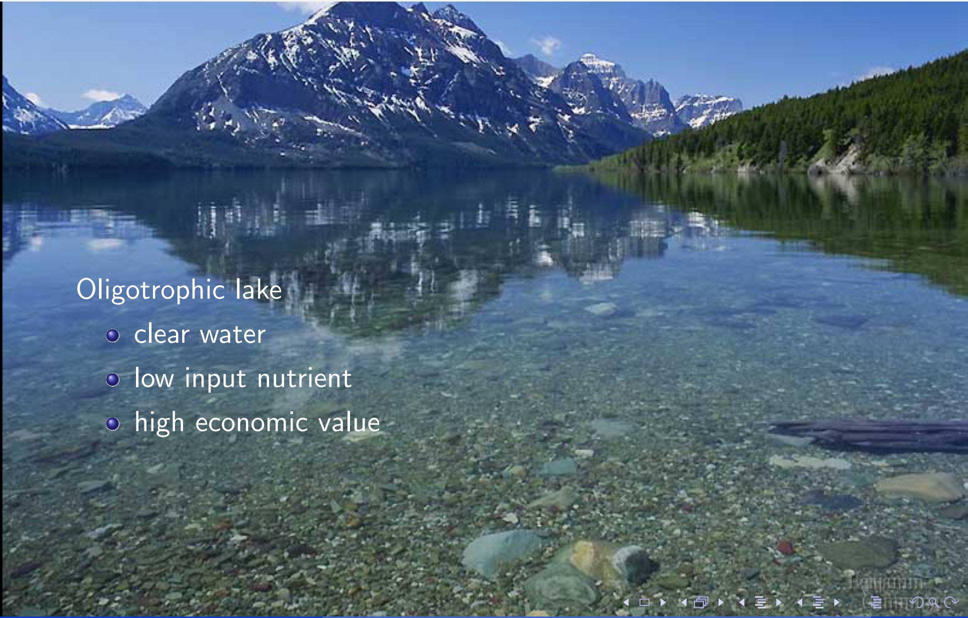
Outline



Oligotrophic lake

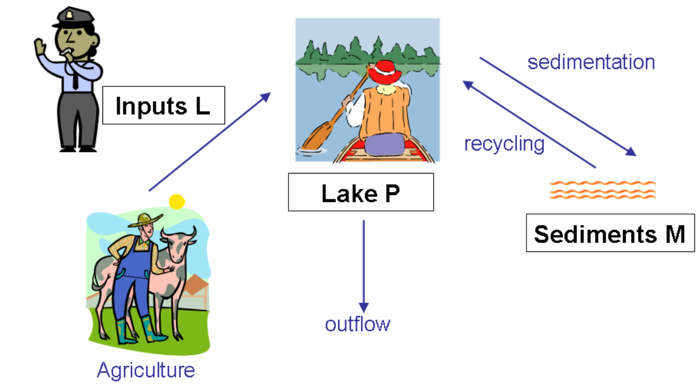
- clear water
- low input nutrient
- high economic value

Eutrophic lake

- turbid water
- high input nutrient
- low economic value

Phosphorus P in the lake

- is the most critical nutrient
- used by the farmers in form of fertilizer or animal feed supplements
- excess P accumulates in the soil and is transported to the lakes



$$x'(t) = \begin{cases} L'(t) = u, \; u \in [-VL; +VL] \\ P'(t) = -(s+h)P(t) + L(t) + rM(t)f(P(t)) \\ M'(t) = -kM(t) + sP(t) - rM(t)f(P(t)) \end{cases} \qquad (1)$$

Property of interest

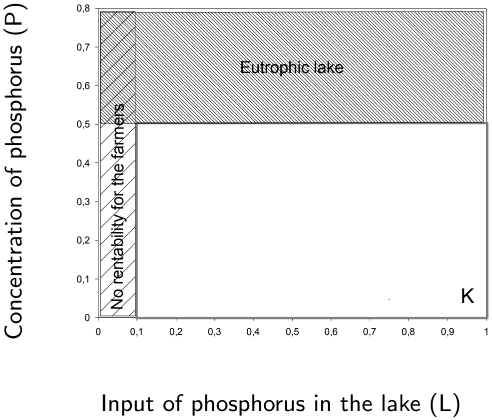
- the lake must remain in an oligotrophic state (population point of view)
  $P \in [0; P_{max}]$
- the profitability of the farmers activities must be ensured
  $L \in [L_{min}; L_{max}]$

We evaluate the resilience of this property of interest

Outline

Viability kernel

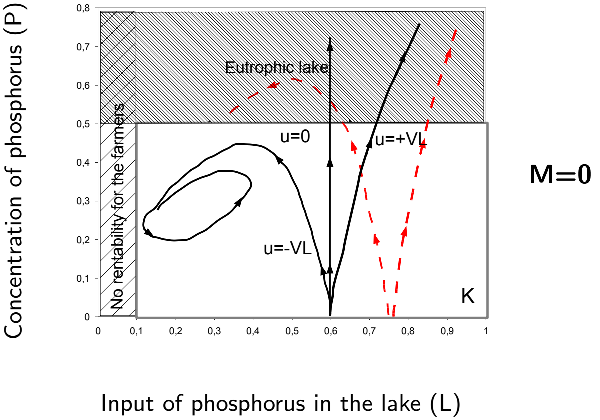
- Aim: define levels of P, M and L that are compatible with the objective to maintain the property of interest



M=0

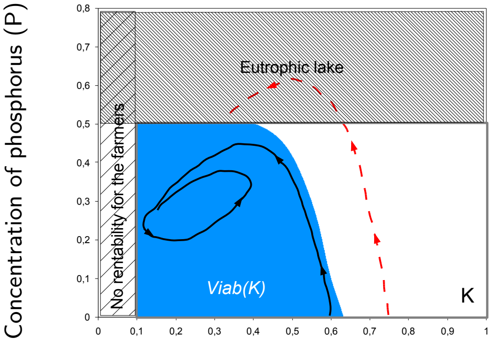
Viability kernel

- Viable state: there exists at least one evolution which allows staying in the viability constraint set



Viability kernel

- Viability kernel: set of all viable states = states for which the property of interest can be maintained



M=0

Input of phosphorus in the lake (L)

Outline

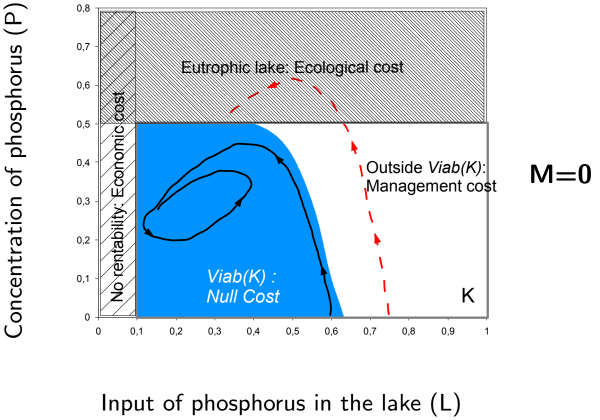
Resilience value computation
Definition

- Resilience: capacity of the system to maintain its property of interest in spite of disturbance
- Martin proposed a mathematical interpretation of resilience

S. Martin
The cost of restoration as a way of defining resilience: a viability approach applied to a model of lake eutrophication.
Ecology and Society, 9(2), 2004.

  - Resilience: inverse of the cost of restoration of the property of interest
  - Based on the viability theory framework

- Viability kernel is the 0-level of the cost function



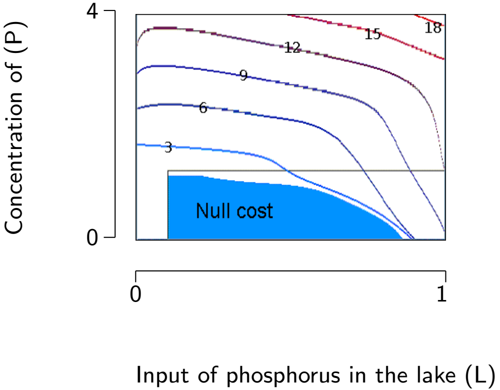
Resilience value computation
Methods

Algorithm to compute resilience values

- Approximating viability kernel algorithm can be used to compute resilience values
- Use a classification method: Support Vector Machines
- We propose a new algorithm that
  - deals with more realistic systems
  - allow to introduce uncertainties on the parameters

Resilience value computation
Restoration costs

- Starting from a non-viable state, the system is doomed to leave K and we look for policies that bring back the system inside $Viab(K)$

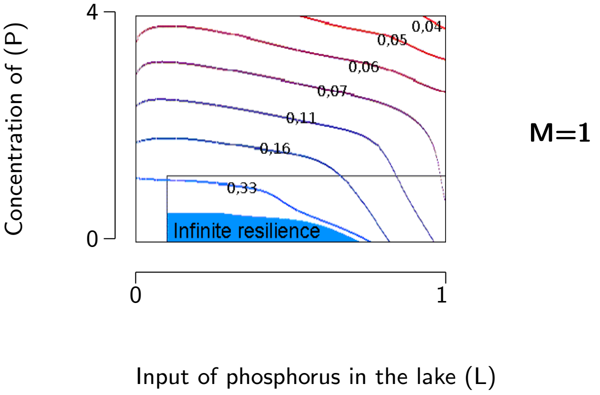


M=1

Resilience value computation

Resilience

- Inverse of the cost to restore the property of interest, lost due to exogenous disturbances
- Maximal disturbance: jump of magnitude $P = 0.5$



Outline

Summary

- Resilience can be defined thanks to viability theory
- We propose a new algorithm that enhances the potential of the approach
- Resilience values allow to define sustainable policies, with the minimal cost