Defining Yield Policies In A Viability Theory Approach

Laetitia Chapel[1], Guillaume Deffuant[1] , Sophie Martin[1] and Christian Mullon[2]

[1]Cemagref, LISC, 24 avenue des Landais, 63172 Aubière Cedex, France, guillaume.deffuant@cemagref.fr

[2]IRD, GEODES, France, christian.mullon@wanadoo.fr

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Introduction

Within the Southern Benguela ecosystem, five different groups (detritus, phytoplankton, zooplankton, pelagic fish and demersal fish) were considered by (Mullon *et al*., 2004) in a dynamical model of biomass evolution. They studied this model in a viability perspective, trying to assess, for a given constant yield, whether each species biomass remains inside a given interval, taking into account the uncertainty on the interaction coefficients. Instead of studying the healthy states of this marine ecosystem with a constant yield, we focus here on the yield policies which keep the system viable. Using the mathematical concept of viability kernel (Aubin, 1991), we examine how yield management might guarantee viable fisheries. Numerical simulations are provided to illustrate the main findings.

Results and discussion

<u>The viability model the Southern Benguela ecosystem</u>

Following a classical approach (Walters *et al*., 1997), we suppose that the biomass flux between a recipient species i and a donor species j depends linearly on the recipient and donor biomasses, with respective coefficients r_{ij} and d_{ij}:

$$\frac{dB_i(j \to i)}{dt} = r_{ij}B_i + d_{ij}B_j$$

This leads to the following expression of the global flow between species:

$$\frac{dB_i}{dt} = \sum_{j \neq i} \left(g_i r_{ij} - d_{ji}\right)B_i + \sum_{j \neq i} \left(g_i d_{ij} - r_{ji}\right)B_j - Y_i$$

where B_i is the biomass of species i, g_i is the growth efficiency of species i, Y_i is the yield of species i.

Mullon *et al*. (2004) take into account the uncertainty on parameters r_{ij} and d_{ij}, which is expressed by:

$$r_{ij} \in \left[\bar{r}_{ij} - \delta r_{ij}, \bar{r}_{ij} + \delta r_{ij}\right], d_{ij} \in \left[\bar{d}_{ij} - \delta d_{ij}, \bar{d}_{ij} + \delta d_{ij}\right].$$

To guarantee a perennial system, the viability constraints are defined by:

$$0 \leq m_i \leq B_i \leq M_i,$$

where m_i is the minimum level for the resource and M_i the maximal biomass which can be contained in the ecosystem.

The controls are the yield on the different species, Y_i.

We use the evaluation of the parameters provided in (Mullon *et al*., 2004).

The viability analysis

The viability theory (Aubin, 1991) aims at controlling dynamical systems with the goal to maintain them inside a given set of admissible states, K, called the viability constraint set. A state is called *viable* if, starting from this state, there exists at least one evolution which remains in K indefinitely. The v*iability kernel* is the set of all viable states and is denoted Viab(K). Aubin (1991) proved the viability theorems which enable to determine the viability kernel, without considering the combinatorial exploration of control actions series. These theorems also provide the control functions that maintain viability.

Such problems are frequent in ecology or economics, when the systems die or badly deteriorate when they leave some regions of the state space. For instance Béné *et al*. (2001) studied the management of a renewable resource as a viability problem. They pointed out irreversible overexploitation configurations related to the resource extinction. Bonneuil (2003) studied the conditions the prey-predator dynamics must satisfy to avoid the extension of one or the other species as a viability problem.

Numerical simulations

To approximate the viability kernels, we use a new algorithm which is built on previous work from (Saint-Pierre, 1994). Its main characteristic is to use an explicit analytical expression of the viability kernel approximation, in order to make it possible to use standard optimisation methods to compute the control. This analytical expression is provided by a learning procedure, the support vector machines (SVMs) (Vapnik, 1998, Cristianini & Show-Taylor, 2000)

This algorithm is interesting in the case we study, because the analytical expression of the viability kernel allows us to use optimisation techniques in high dimensional control spaces.

From a non viable point, one could compute the yield policy which allows to come back to a viable state in a minimum time, or minimising some additional cost, which corresponds to the definition of the resilience proposed in (Martin, 2004).

Conclusion

Solving the viability problem provides all yield policies if any which guarantees a perennial system.

One of the main practical difficulties up to now with the viability theory was the lack of methods to solve the problem in large dimensions. The use of learning procedures such as SVMs gives this theory a larger practical potential.

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