Modeling the 802.11 Distributed Coordination Function in Non-saturated Conditions

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Abstract— **Analysis of the 802.11 CSMA/CA mechanism has received considerable attention recently. Bianchi [1] presents an analytic model under a saturated traffic assumption. Bianchi's model is accurate, but typical network conditions are non-saturated. We present an extension of his model to a non-saturated environment. Its predictions are validated against simulation and are found to accurately capture many interesting features of non-saturated operation.**

Keywords— **Wireless LAN, IEEE 802.11 MAC, Non-saturated Traffic, Performance Evaluation.**

I. Introduction

The 802.11 wireless LAN standard has been widely deployed during recent years and has received considerable research attention. The 802.11 MAC layer uses a CSMA/CA algorithm with binary exponential back-off to regulate access to the shared wireless channel. While this algorithm has been the subject of numerous empirical studies, an analytic framework for reasoning about its properties remains notably lacking. Developing analysis tools is desirable not only because of the wide deployment of 802.11 equipment but also because the CSMA/CA mechanism continues to play a key role in new standards proposals such as 802.11e. A key difficulty in the mathematical modeling the 802.11 MAC lies in the very large number of states that may exist (scaling exponentially with the number of nodes). In his seminal paper, Bianchi [1] addressed this difficulty by assuming that (i) every node is saturated (i.e. always has a packet to be transmitted) and (ii) the packet collision probability is constant regardless of the state or station considered. Provided that every node is indeed saturated, the resulting model is remarkably accurate. Unfortunately, the saturation assumption is unlikely to be valid in most real 802.11 networks. Data traffic such as web and email is typically bursty in nature while streaming traffic such as voice operates at relatively low rates and often in an on-off manner. Hence, for most real traffic the demanded transmission rate is variable with significant idle periods. Our aim is to derive a mathematical model that relaxes the restriction to saturated operation while retaining as much as possible of the attractive simplicity of Bianchi's model (in particular, the ability to obtain analytic relationships).

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II. Analysis

Bianchi [1] presents a Markov model where each station is modeled by a pair of integers (i, k). The back-off stage, i, starts at 0 at the first attempt to transmit a packet and is increased by 1 every time a transmission attempt results in a collision, up to a maximum value m. It is reset after a successful transmission. The counter, k is initially chosen uniformly between $[0, W_i - 1]$, where $W_i = 2^i W_0$ is the range of the counter. While the medium is idle, the counter is decremented. Transmission is attempted when $k = 0$.

We assume each station can buffer one packet and there is a constant probability q of at least one packet arriving per state. Thus we introduce states $(0, k)_e$ for $k \in [0, W_0 - 1]$, representing a node which has transmitted a packet, but has none waiting. Note that $i = 0$ in all such states, because if $i > 0$ then a collision has occurred, so we must have a packet awaiting transmission.

We now derive relationships between: p, the probability of collision; P, the Markov chain's transition matrix; b, the stationary distribution; and τ, the transmission probability per station. These relationships can be solved for p and τ, and network throughput predicted. It is important to note that the Markov chain's evolution is not real-time, and so the estimation of throughput requires an estimate of the average state duration.

The simplest transitions are those where the counter is nonzero. If we have a packet, then the only possible change is that the counter decrements. If we do not have a packet, the counter will decrement, but a packet may also arrive with probability q. Thus, for $0 < k < W_i$ we have

$$\begin{aligned} 0 < i \leq m, \quad P[(i, k-1)|(i, k)] &= 1, \\ P[(0, k-1)_e|(0, k)_e] &= 1 - q, \\ P[(0, k-1)|(0, k)_e] &= q. \end{aligned}$$

If the counter reaches 0 and a packet has arrived, we begin a transmission. We assume there is a probability p that another node transmits at the same time, resulting in a collision and an increase in the back-off stage. Thus for $0 \leq i \leq m$ and $k \geq 0$ we have

$$\begin{aligned} P[(0, k)_e|(i, 0)] &= \frac{(1-p)(1-q)}{W_0}, \\ P[(0, k)|(i, 0)] &= \frac{(1-p)q}{W_0}, \\ P[(\min(i+1, m), k)|(i, 0)] &= \frac{p}{W_{\min(i+1,m)}}. \end{aligned}$$

The most complex transitions are from the $(0, 0)_e$ state, where the countdown is complete, but we have no packet to send. If no packet arrives, we stay in this state. If a packet arrives, the new state depends on the current state of the medium: if the medium is idle we may begin transmission, which may result in a successful transmission or a collision; if the medium is busy, the 802.11 MAC begins another stage-0 back-off. This gives

$$\begin{aligned} & P[(0,0)_e|(0,0)_e] &=& 1-q+\tfrac{q(1-\tau)^{n-1}(1-p)}{W_0}, \\ k>0, & P[(0,k)_e|(0,0)_e] &=& \tfrac{q(1-\tau)^{n-1}(1-p)}{W_0}, \\ k\geq 0, & P[(1,k)|(0,0)_e] &=& \tfrac{q(1-\tau)^{n-1}p}{W_1}, \\ k\geq 0, & P[(0,k)|(0,0)_e] &=& \tfrac{q(1-(1-\tau)^{n-1})}{W_0}. \end{aligned}$$

Note that we have used $(1-\tau)^{n-1}$ as the probability that the medium is idle. As noted by Bianchi, $1-p=(1-\tau)^{n-1}$, thus our transition probabilities only depend on p and q.

Solving for the stationary distribution, b, yields (after lengthy algebra)

$$\begin{aligned} 1/b_{(0,0)_e} &= (1-q)+\frac{q^2W_0(W_0+1)}{2(1-(1-q)^{W_0})} \\ &+\tfrac{q(W_0+1)}{2(1-q)}\left(\tfrac{q^2W_0}{1-(1-q)^{W_0}}+p(1-q)-q(1-p)^2\right) \\ &+\tfrac{pq^2}{2(1-q)(1-p)}\left(\tfrac{W_0}{1-(1-q)^{W_0}}-(1-p)^2\right) \\ &\qquad\left(2W_0\tfrac{1-p-p(2p)^{m-1}}{1-2p}+1\right) \end{aligned} \quad (1)$$

and

$$\begin{aligned} \tau &= \textstyle\sum_{i=0}^{m} b_{(i,0)}+b_{(0,0)_e}q(1-p) \\ &= b_{(0,0)_e}\tfrac{q^2}{1-q}\left(\tfrac{W_0}{(1-p)(1-(1-q)^{W_0})}-(1-p)\right). \end{aligned} \quad (2)$$

For given values of q, W_0, n and m we may solve (2) against $1-p=(1-\tau)^{n-1}$ to determine p and τ. In the limit $q\to 1$, our model yields the same value for τ and p as Bianchi's saturated model.

The expression for throughput is the same as in [1],

$$S=\frac{P_sP_{tr}E}{(1-P_{tr})\sigma+P_{tr}P_sT_s+P_{tr}(1-P_s)T_c},$$

where $P_{tr}=1-(1-\tau)^n$, $P_s=n\tau(1-\tau)^{n-1}/P_{tr}$, E is the time spent transmitting payload data, σ is the time for the counter to decrement, T_s is the time for a successful transmission and T_c is the time for a collision. The denominator of this fraction is the expected duration of a state in the Markov chain in real-time, which we denote T.

To match the model with experiment we must relate q to offered packet load. Modeling a saturated system, i.e. there is always a packet awaiting service, is achieved by setting $q=1$. If packets arrive in a Poisson manner with exponentially distributed inter-packet arrival times with rate λ, then $1-q$ is the probability no packet arrives in a typical slot of length T. That is $1-q=\exp(-\lambda T)$ and therefore $q=1-\exp(-\lambda T)$.

III. Validation

The model was verified against TU-Berlin's [2] ns2 802.11 simulator. MAC parameter values (corresponding to 802.11b) and packet sizes used are in Table I. Varying numbers of stations, with a small buffer, were simulated. In the first set of simulations, arrivals to each station are Poisson.

W_0	31	E	407us	T_s	986us
m	5	σ	20us	T_c	986us

TABLE I

Parameters values for model and simulation.

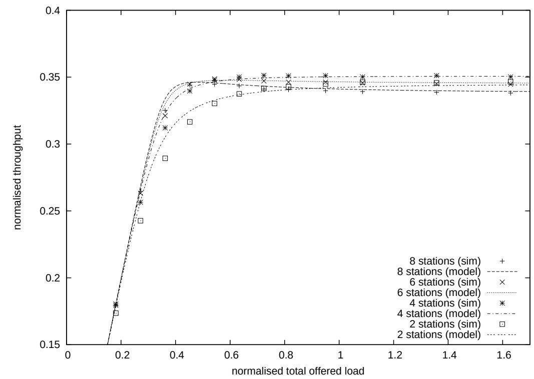

Fig. 1. Throughput vs. offered load for small numbers of nodes. For rates below those shown, there is agreement.

Figures 1 and 2 show predicted and simulated throughput against offered load, for a different numbers of wireless nodes (arrival rates are normalized by the data rate of 11Mbs). Collision probabilities corresponding to Figure 1 are shown in Figure 3 (similar accuracy is obtained for the conditions used in Figure 2). The model accurately captures important features. In particular,

- the linear relationship (with slope 1) between throughput and offered load under low loads.
- the limiting behavior of throughput at high offered loads (corresponding to saturation).
- the complex transition between under-loaded and saturated regimes is accurately captured. For small numbers of nodes, saturation throughput is the peak throughput. For larger numbers of nodes, the throughput falls as we approach saturation and peak throughput is achieved *before* saturation. The offered load at which this peak occurs is relatively insensitive to the number of nodes.

In the foregoing plots, packets arrivals are Poisson, yield-

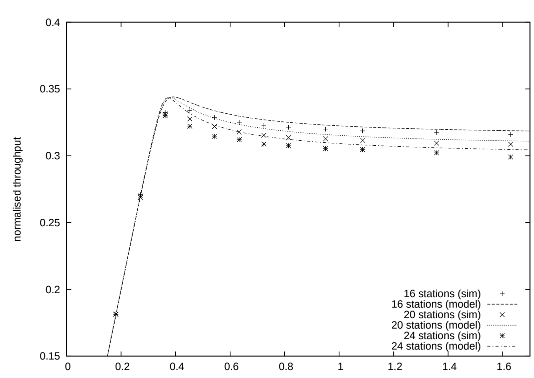

Fig. 2. Throughput vs. offered load for larger numbers of nodes. For rates below those shown, there is agreement.

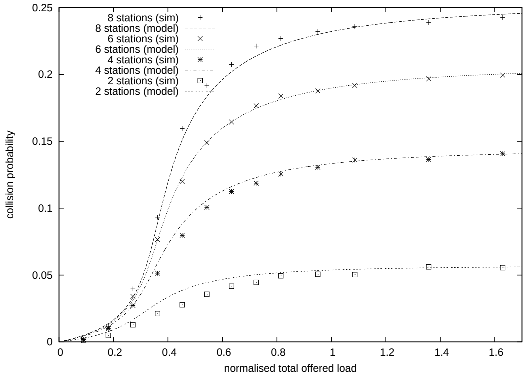

Fig. 3. Collision probability vs. offered load for small numbers of nodes.

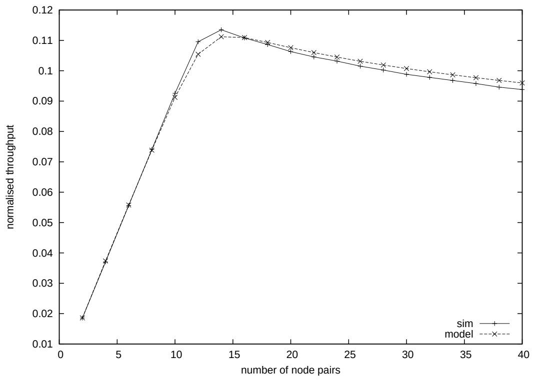

Fig. 4. Throughput vs. numbers of node-pairs sending 64kbps on-off traffic streams.

ing independent arrivals at a specified mean rate. However, we have found that similar results hold for a range of traffic types. To illustrate this, we briefly present results for simulated voice traffic with silence suppression. Following [3], we generate a 64kbs on-off traffic stream with on and off periods exponentially distributed with mean 1.5s, subject to a minimum of 240ms. Traffic is between pairs of nodes to account for the two-way correlated nature of voice conversations; the on/off periods of one node correspond to the off/on periods of another. We apply our model to node-pairs when making predictions. Predicted and simulated throughput versus the number of node-pairs are shown in Figure 4, where it can be seen that our model is remarkably accurate.

IV. Considerations

It is easy to consider small variations on this model, such as disallowing packet arrival immediately after transmission, ignoring carrier sense in state $(0,0)_e$, or by limiting the number of retransmission attempts. We have investigated these possibilities and found that they result in numerical changes that are not significant.

Two important assumptions of the model are constant probability of arrival per state and small interface buffers. The accuracy of the model predictions for a range of traffic types, as noted previously, suggests there is a useful robustness with respect to the first assumption. We have found that the predictions are more sensitive to the presence of large buffers. It is possible to introduce extra states to model longer queues, and also to allow variable packet arrival probabilities per state. Owing to space restrictions this is beyond the scope of this paper.

V. Related Work

There are alternative approaches to non-saturated modeling. In [4] a modification of [1] is considered where a probability of not transmitting is introduced that represents a station having no data to send. The model is not predictive as this probability is not known as a function of load and must be estimated from simulation. In [5] idle states are added after packet transmission to represent bursty arrivals in a way that does not account for post-backoff. In [6] a model where states are of fixed real-time length is introduced, but does not capture the feature of a pre-saturation throughput peak. In [7] a model incorporating postbackoff is presented, but not solved explicitly. In [8] a non-Markov model is developed, but is based on an unjustified assumption that the saturated setting provides good approximation to certain unsaturated quantities.

VI. Conclusion

We present a model of the 802.11 MAC layer in non-saturated conditions. It is analytically tractable yet remarkably powerful. It is shown to be in quantitative and qualitative agreement with detailed simulations, yielding accurate predictions of throughput and collision probability. It captures important features of non-saturated operation (e.g. throughput may be higher in non-saturated operation than saturated). It is accurate for a range of traffic types. This is illustrated with voice calls (to the authors' knowledge this is the first demonstration of an analytic model of voice calls in 802.11). The model is interesting not only because of wide deployment of 802.11 equipment and prevalence of non-saturated operation in wireless networks, but also because the CSMA/CA mechanism plays a central role in new standards such as 802.11e [9].

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