ANALYSIS OF ROBUST PERFORMANCE IN INTERCONNECTED SYSTEMS USING THE STRUCTURED SINGULAR VALUE

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This paper presents new analysis techniques that provide 'limit of performance' answers to the study of interconnected multivariable systems. As an example, the question of worst case performance for an automatic suspension system is considered. Performance in this paradigm is defined as the minimisation of the effect of output disturbances on passenger comfort despite the constraints of uncertain vehicle mass, centre of gravity, variable external road surfaces etc. An engineer would normally approach a problem of this type by first working with a low order model that would reasonably capture some of the fundamental dynamic characteristics of the system and hence implement a suitable control law. Subsequent iterations of the design cycle would result in the model being extended (and perhaps even ultimately replaced) when it was no longer capable of satisfying required performance objectives.

In this paper a first principles representation of an active suspension system is presented that reduces to a simple double mass-spring-damper model. This is perhaps the simplest example of an 'interconnected' system, one where dynamic behaviour at the output is the product of a 'system' of systems. This paper demonstrates that some non-trivial robust performance analysis questions exist, even for this deceptively simple model. These questions have significant implications regarding the analysis of more realistic, i.e., higher order, possibly nonlinear, systems that may be required to adequately reflect the dynamic behaviour of the actual system. There are surprisingly few practical robustness analysis examples in the literature where these subtle analysis questions can be addressed in an accessible, rigorous and easily reproducible fashion.

This work considers the use of the structured singular value, μ, on this example and illustrates how the analysis question can become quite complex through the addition of quite intuitive physical parameters to the model that are uncertain, but are only allowed to vary in a constrained fashion. Significant computational difficulties with this type of problem become quickly apparent to many first-time users of the MATLAB μ-Tools Toolbox where the lower bound algorithm of [1] fails to converge when the uncertainty set is constrained to be real. This is a major cause of concern, mainly because no useful information about the vector of physical parameters that causes worst case performance to occur is then available. In the authors' experience this can generate significant doubts for the student who is new to the subject as to the efficacy of the machinery that underpins the whole methodology.

Here, the so-called "μ-paradigm" for the computation of the worst case (or maximum gain) in an uncertain low order interconnected model is considered critically. In particular, the merits and drawbacks for an analysis question where the uncertainty is quite naturally constrained to be strictly real is discussed. In this fashion quite subtle limitations in existing μ-analysis software are highlighted quickly in an accessible manner. Moreover, conventional analysis techniques based on Gain or Phase margin are shown to be poor indicators of robustness. A *certifiably* safe method for the computation of worst case performance is presented that transforms the problem to include frequency as a real uncertain parameter in the analysis question. Computation reduces to a single (so called "skew") μ calculation on a static matrix that is inherently attractive. An optimisation-based skew μ algorithm developed in [2] is used to determine good lower bound information. It is shown how by using this approach a perturbation is returned that provides useful lower bound information for the engineer over a user defined frequency interval. The skew μ upper bound algorithm presented in [3] is used to determine whether any solution found using the lower bound algorithm is local or global, and to ensure that no higher value of μ exists over an entire frequency range.

REFERENCES

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