Complex Embedded Automotive Control Systems
CEmACS

DaimlerChrysler
SINTEF
Glasgow University
Hamilton Institute
Lund University

Public
Interim Report Workpart 3
Deliverable D9

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Executive Summary

This is the interim report of work package 3 *Controller Design* of the CEMACS project (deliverable D9). It covers the first 12 months of activities towards Milestone 2.

The focus of Workpart 3 is on the theory and methods of controller design with the aim to adapt these to the applicational needs of the project. Most project partners have been active in this work package during the reporting period. This can be summarised as follows: SINTEF has worked on WP 3.2 (Hybrid Control Systems) and WP 3.4 (Nonlinear and Adaptive Control), Glasgow University has been active on WP 3.1 (Multivariable Control), the Hamilton Institute has contributed to WP 3.2 and 3.4, while Lund University has worked on WP 3.1, 3.2, 3.3 (Multivariable Control Systems with time delay) and 3.4.

The expected result for Milestone 2 is an integrated set of design methods covering all areas of the work package. Based on the complementary set of design tools the consortium has developed design strategies for the application work packages.

Consequently, this report presents the design methods as they relate to the applications in the work packages 1 (Vehicle control for active safety) and 2 (Integrated Chassis control / Generic prototype).

Contents

1 Vehicle control for active safety

Workpackage 1 is concerned with the evaluation of novel multivariable control and nonlinear state observation approaches to stabilise cars close to the physical limits for increased active safety. Two applications are considered, roll-over protection and collision avoidance.

1.1 Multivariable controller design for collision avoidance

Design tools based on Individual Channel Analysis and Design (ICAD) (O'Reilly and W. E. Leithead 1991, O'Reilly and Leithead 1995) are used for to develop controllers for collision avoidance. Further details are given in the case study presented in section 3 of this report.

1.2 Dynamic Control Allocation for Yaw Stabilization of an Automotive Vehicle.

Work has continued to establish novel hydrid control design methods, and qualify recently developed advanced hydrid control methods for application in automotive applications. Recently developed numerical methods for multi–parametric nonlinear programming have been tested on an automotive yaw stabilization problem, providing similar functionality as ESP (electronic stability program). The method gives an explicit characterization and representation of the optimal solution, and is promising for handling of complex, nonlinear and dynamic constraints when coordinating a set of automotive brake and steering actuators (Tøndel and Johansen 2005*a*). Full details are given in the case study presented in section 4 of this report.

1.3 Multiple Model estimation of Center of Gravity

We have started to look at the *Multiple-Model Adaptive Control* paradigm for applications on automotive control problems. In particular we have looked at Multiple Model estimation of Center of Gravity (CG) Height problem for use in vehicles with high center of gravity and we started analyzing some theoretical issues related to it. We have tested and verified the idea with CASCADE simulations using a compact car (A-Class) and experimental data for an SUV (M-Class). We started looking at the Anti-Rollover controller based on differential braking integrated with the CG height estimation algorithm, and preliminary results proved that we can get better performance than a robust design based on constant feedback gain. We also started looking at the pitch and longitudinal dynamics of the car and we aim to have an alternative method for estimating CG height during acceleration and deceleration of the car, thus covering the whole operating conditions.

Dissemination: Solmaz, S., Akar, M., Shorten, R., 'Multiple Model CG height Estimation for SUV Rollover Prevention', Patent application in preparation.

Solmaz, S., 'Multiple-model estimation and control - A review', Hamilton Institute Technical Report.

A. Paul, M. Stefanovic, M. G. Safonov and M. Akar, "Multiple controller adaptive control for a tracking problem using an unfalsification approach", to appear in Proc. CDC/ECC'05, Seville, Spain, December 2005.

M. Akar, 'Adaptive robust tracking of nonlinear systems using multiple controllers and switching', submitted to Systems & Control Letters.

1.4 Dynamic programming for analysis of roll-over protection

Synthesis methods for nonlinear control are developed based on classical ideas of dynamic programming in a modern computational setting based on convex optimization. The same computational methods are applied to analysis of roll-over protection The dynamic programming methods, including Matlab implementations, developed by Rantzer and Wernrud (Wernrud and Rantzer 2005) will be applied in order to find bounds on the maximal stabilizable region of the vehicle state space. In other words, to answer the question: Under what circumstances is it possible to prevent the vehicle from rolling over?

1.5 Robust stability in the presence of bounded uncertain time-varying time delays

Robust stability of linear systems in presence of bounded uncertain time-varying time delays is studied. This is relevant for network delays in the control system of a modern car. The time delay robustness problem is treated in the Integral Quadratic Constraint framework. The stability criterion is formulated as a frequency dependent linear matrix inequality, or equivalently as a Semi-Definite Program (SPD). Therefore, the criterion can be checked efficiently by using various SDP solvers (Kao and Rantzer 2005, Kao and Lincoln 2004).

The analysis of robustness to time-delays will be applied to any particular control design before implementation in the car, to limit performance deterioration due to delays on the communication bus.

1.6 Decentralization of control

A central theoretical issue is decentralization. This is important in car dynamics, because the fast loops need to operate locally, while coordination and parameter estimation can work on a slower time scale. We work on decentralized solutions in state observers and observer based feedback. We also study how experiences from industrial success of PID control can be transfered to a multi-variable setting.

2 Integrated Chassis control / Generic prototype

Workpackage 2 is concerned with the evaluation of advanced control methods for the emulation of vehicle dynamics in experimental test vehicles. The design framework focusses on multivariable and hybrid control methods.

Issues of stability in the presence of uncertain time-varying time delays as discussed in section 1.5 and decentralization of control (cf. section 1.6) apply here as well.

2.1 ICAD and QFT to control lateral dynamics in a 4-wheel steering system

We have applied ICAD and QFT methods to the problem of controlling the lateral dynamics of a 4-wheel steering system. We have also estimated the emulation envelope of the test vehicle Pegasos and have investigated the interactions between lateral and vertical dynamics. Initial results have been documented and publications are planned.

Further, Pegasos is equipped with a hydro-pneumatic actuator in the vehicle suspension. We have carried out initial tests on this actuator with a view to verify the DaimlerChrysler simulation model of Pegasus. Initial results suggest that the hydro-pneumatic system has yet to be adequately modelled. This will form part of the work-programme for the next reporting period.

Dissemination: Leith, D.J., Leithead, W.E., Vilaplana, M. 'Robust lateral controller for 4-wheel steer cars with actuator constraints', Proc. of IEEE Conference on Decision and Control, Bahamas, 2004.

Report on QFT method applied to 4-Wheel steering problem, In preparation, C. Villegas, M. Barreras, J. Kalkkuhl.

2.2 Hybrid control in a 4-Wheel steering system.

Work on hybrid control has progressed along several lines of enquiry. Basic work has involved investigating the quadratic and nonquadratic stability of switched linear systems. We have also investigated the relationship between stability/performance and controller realisation for a class of switched linear systems, and developed techniques for the design of switched linear systems with constraints.

Further work is progressing on verification of safety properties and optimization of switch strategies. In both cases, methods from discrete automata are generalized to work for a general class of hybrid systems (Rantzer 2005).

Hybrid control methodologies are being applied to the control of the lateral dynamics in a 4-wheel steering system.

Dissemination: We have presented lectures of this material at: (i) University of Limerick; (ii) Purdue; (iii) UC San Diego; (iv) ACC (Portland); and (v) CDC Seville (forthcoming). We also presented a course in this area at the 2005 ACC (Portland). Dr. Shorten is a guest editor of the forthcoming special issue of the IEE Proceedings on control theory on this topic.

R. Shorten and C. King, 'Spectral Conditions for Strict Positive Realness', IEEE Transactions on Automatic Control, October 2004.

R. Shorten and O. Mason, 'The geometry of convex cones associated with the Lyapunov inequality and the common Lyapunov function problem', Accepted for Electronic Linear Algebra, 2005.

Sun, Z. and Shorten, R., 'On convergence rates for switched triangular systems', Provisionally Accepted for publication in IEEE Transactions on Automatic Control, 2005.

King, C., Shorten, R. ‘A spectral condition for the existence of a CQLF’, Accepted for publication in Linear Algebra and its Applications, 2005.

O. Mason, E. Zeheb, S. Solmaz, R. Shorten, ‘Some results on quadratic stability of switched systems with interval uncertainty’, Submitted to IEEE Transactions on Circuits and Systems: II.

O. Mason, E. Zeheb, S. Solmaz, R. Shorten, ‘On the quadratic stability of switched interval systems: Preliminary results’, Intelligent Control, 2005. Proceedings of the 2005 IEEE International Symposium on, Mediterranean Conference on Control and Automation 27-29 June 2005 Page(s):12 - 17

O. Mason, S. Solmaz, R. Shorten, ‘A Generalized Matrix Inertia Result for the Existence of Common Quadratic Lyapunov Functions for a Pair of Matrices with Rank-1 Difference and Same Regular Inertia’, Submitted for Linear Algebra and its Applications.

E. Zeheb, R. Shorten, ‘Eigenvalue criteria for strict positive realness and related results’, Submitted to IEEE Transactions on Automatic Control.

K. Wulff, R. Shorten, F. Wirth, ‘On the stabilization of switched systems defined by N linear SISO systems”, CDC/ECC, 2005.

King, C. and Shorten, ’On the Design of Stable State Dependent Switching Laws for Single-Input Single-Output Systems’, CDC/ECC 2005.

Shorten, R., Wirth, F. Mason, O., King, C. and Wulff, K. ‘Stability theory for switched and hybrid systems’, Submitted to SIAM Review.

Mason, O., Shorten, R. ‘On the simultaneous diagonal stability of a pair of positive linear systems’, Accepted for publication in Linear Algebra and its Applications, 2005.

3 Case study: Multivariable controller design for collision avoidance

3.1 Introduction

This section of the report presents aspects of non-linear control design for collision avoidance; it is motivated by the work being undertaken in work package 1.2. A specification for the overall controller performance is provided in interim report D7. The controller will cause a passenger vehicle to perform a lateral manoeuvre at its physical limits. The vehicle dynamics are complex and highly non-linear. Furthermore, it will be necessary to control multiple inputs and outputs simultaneously to achieve tracking of a target trajectory while maintaining directional control. Thus the system would seem to be an appropriate problem to further development of the multivariable control technique of individual channel analysis and design (ICAD (O'Reilly and Leithead 1991)).

3.2 High level controller architecture

The high level conceptual architecture for a collision controller is shown in figure 1. A pre-defined reference trajectory is selected by the vehicle computer according to the environmental conditions in which the vehicle finds itself when a collision avoidance manoeuvre is initiated.

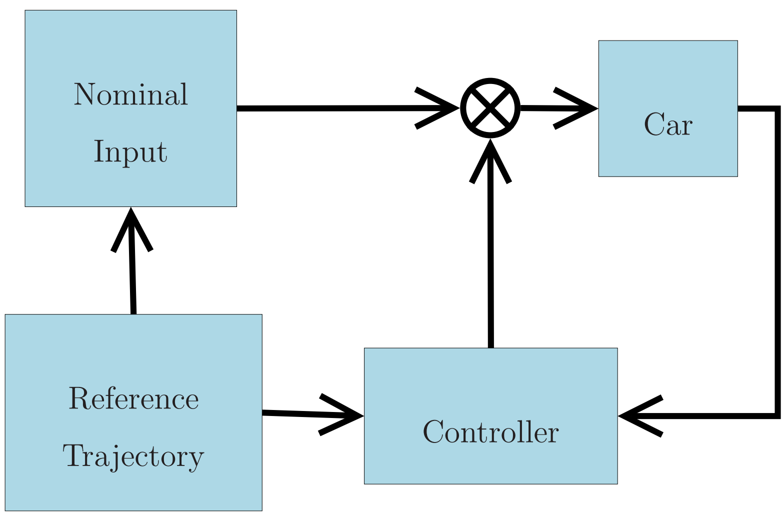


Figure 1: Conceptual architecture

From the reference trajectory, nominal inputs can be calculated to cause the vehicle to steer a course that follows the reference. Appropriate control inputs can be obtained from an inverse vehicle model and knowledge of the prevailing environmental conditions. Applying these inputs to the vehicle will cause it to follow approximately the required course. A feedback controller can then account for errors due to disturbances, uncertainties and unmodelled dynamics.

In particular, the feedback controller will act to stabilise the vehicle on its trajectory and cause it to remain at, or within, the physical limits of its performance.

3.3 System model

An appropriate system model is required on which to base the design of the controllers. The system (a passenger vehicle performing an abrupt lateral manouevre) will be required to operate at its physical limits in a highly non-linear region of its dynamic envelope and far from equilibrium conditions. An appropriate choice of model is a velocity-based, two track model. A velocity-based model is valid throughout the entire envelope of the vehicle, not just at equilibrium conditions, and is amenable to smooth gain scheduling to produce a globally valid approximate model (Leith and Leithead 1998). A description of this model is given in deliverable report D1. The source code and documentation for a specific implementation of the model are provided in deliverable report D7.

The plant has five inputs: torque control on each of the four wheels via the anti-lock braking system (ABS) and the steering angle of the front wheels; there are three outputs to be controlled: the vehicle trajectory (longitudinal and lateral position) and the vehicle heading (yaw angle). The actuator characteristics are specified in reports D7 and D11.

Measurements of the outputs and other relevant vehicle states from the global position system (GPS) and inertial sensors will be fused in an observer being developed as part of work package 4.

3.4 Stabilisation of the vehicle on its trajectory with a feedback controller

Several design concepts are under consideration for the low level architecture of the controller. Refinement and evaluation of these concepts will occur in the subsequent phase of the project.

3.4.1 Unity feedback controller

A generic controller with unity feedback is shown in figure 2.

A controller K may be constructed purely from constant terms, resulting in a simple gain matrix. Alternatively, it may be a function of s, the Laplace operator, in which case a PID controller, or something more complicated, may result.

For a linear, multi-input, multi-output (MIMO) system, ICAD can be applied to obtain an appropriate gain matrix which will enable the system to achieve its desired target while minimising adverse interaction between the multiple input-output channels. This technique is well established in the existing literature for linear systems, as well as for non-linear systems for which appropriate linearisations can be obtained (Vilaplana *et al.* 2005, Dudgeon and Gribble 1997).

Emergency collision avoidance requires an aggressive manouevre during which the local system dynamics will be changing rapidly. Classical gain scheduling can

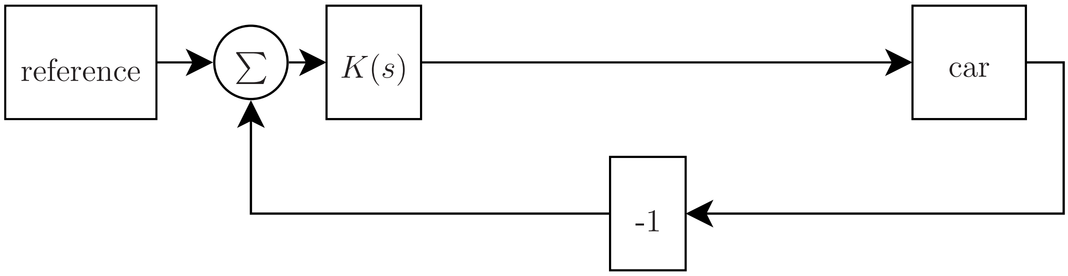


Figure 2: Generic controller with unity feedback

account for the changing plant, however the vehicle will also be taken to its physical limits far from equilibrium conditions, thus reducing the validity of models obtained by linearisation about such operating points. Unfortunately, the technique of gain scheduling is not generally appropriate for systems which will be operating far from equilibrium; blending of the local linearisations does not lead to a smooth, globally valid model. The use of a velocity-based linearisation can overcome this problem (Leith *et al.* 2001) to produce an appropriate model

3.4.2 Inverse controller

An alternative approach to classical gain scheduling is the use of an inverse system model to calculate the correct inputs to cause the vehicle to follow a specified trajectory. A controller structure which uses model inversion is shown in figure 3.

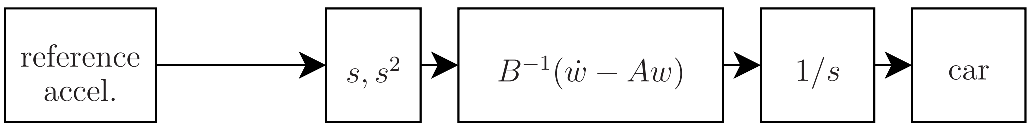


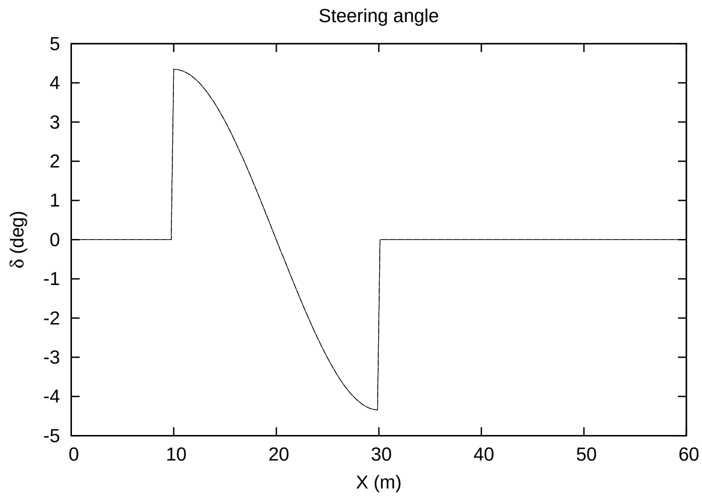
Figure 3: Inverse acceleration controller. **w** is the derivative of the acceleration vector. A and B are the state and input matrices of the velocity-based linearisation of the system.

In this structure, a velocity-based linearisation of the system is used; inversion of a full non-linear system model would create substantial difficulties for subsequent analysis to ensure the safety of the closed loop system. To enable use of the velocity-based linearisation the reference signal is differentiated and the output integrated before being applied to the vehicle.

Inverse controllers can work well in theory, however in practice there are well known limitations.

A significant problem with using pure model inversion is that poorly conditioned system matrices can lead to large actuator demands for those inputs to which the output is relatively insensitive. Using an exact pseudo-inverse of the vehicle model with five actuators (steering angle and four braking torques) can lead to high demanded tyre forces with relatively little steering input; the controller attempts to complete the manoeuvre predominantly by means of differential braking instead of steering. Disabling the use of differential braking, uncorrected model inversion is still unsuitable as a controller. Feedforward controllers based on system inversion often cope poorly with parametric uncertainties and unmodelled dynamics. Additionally, the lack of feedback in this structure leaves little scope for correction of any deviation from the desired trajectory, as demonstrated in figures 4(a) to 5(c).

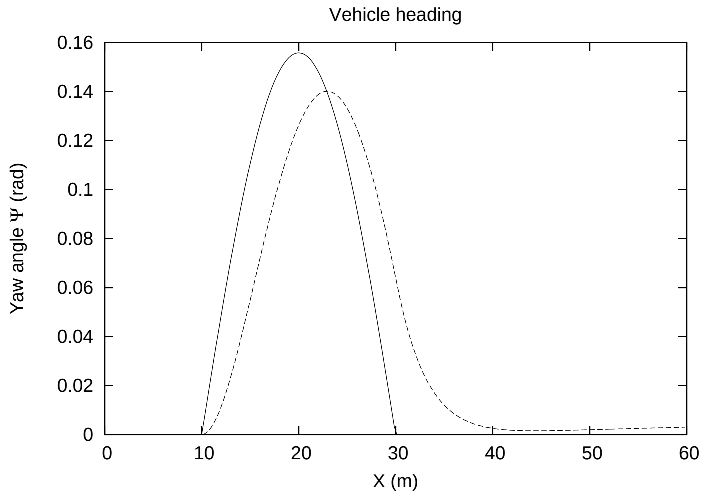
Figure 4(a) shows the desired and actual steering angle during a simulation with the non-linear vehicle model from which the velocity-based linearisation was derived. Control of wheel torques has been disabled in this simulation to make the effects clear. The desired steering angle was obtained by calculating the Ackerman angle using the assumption of steady state cornering. This assumption is not entirely valid for the vehicle conditions in which the manouevre is occurring. Combined with approximations from the linearisation process, this assumption leads to an error in the vehicle yaw, as shown in figure 4(b).



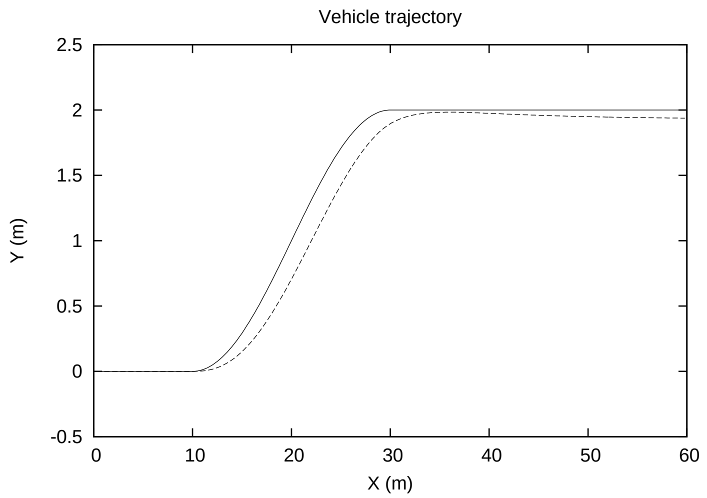
(a) Desired and actual steering angle to perform a lane-change manoeuvre. Control of longitudinal wheel forces has been disabled in this simulation.

Figure 4: Simulation of a lane-change manoeuvre using only steering. The steering angle is set to the Ackerman steering angle required for steady state cornering. Lack of feedback means that there is no opportunity to correct an error in the trajectory due to the yaw rate failing to meet its target. The reference value is depicted by a solid line, the actual value by a dashed line.

As expected, the error in yaw angle leads to an error in the vehicle trajectory, as seen in figure 5(c).



(b) Desired and actual vehicle yaw angle using steering control only.



(c) Desired and actual vehicle lateral position using steering control only.

3.4.3 Combined gain matrix with feedback and inverse control

It is possible that the combination of classical gain matrix with an inverse system model can provide a suitable controller structure for controlling the vehicle during the manoeuvre. An inverted (velocity-based) linearisation of the system provides the benefits of gain-scheduling by adapting to the changing plant dynamics while a conventional gain matrix provides the usual disturbance rejection characteristics.

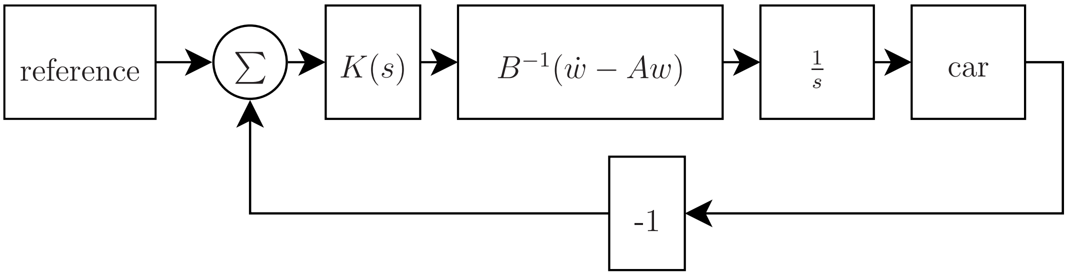


Figure 5: Generic feedback controller structure using the inverse model

Promising results have been seen with such a structure at specific operating points. It remains to apply ICAD over the full operating range of the system to obtain a globally valid controller.

Figures 6(a) to 7(c) show the vehicle performing a similar manoeuvre to that described in the previous section. Again, the commanded steering angle is the Ackerman steering angle. In this case, however, longitudinal wheel forces are allowed to correct for errors in the vehicle trajectory. Appropriate scaling of the channels is accomplished through the K matrix.

An error signal **e** is composed of the errors in the longitudinal and lateral displacement of the vehicle and the yaw rate

$$\mathbf{e} = \begin{pmatrix} X_d - X \\ Y_d - Y \\ \dot{\Psi}_d - \dot{\Psi} \end{pmatrix}$$

The desired rate of change of the derivative of the acceleration (i.e. the rate vector of the velocity-based model) is then chosen to be

$$\dot{\mathbf{w}} = -0.075 \times \mathbf{e}$$

This is equivalent to using an error signal composed of lateral, longitudinal and yaw displacement and a gain matrix K such that

$$\mathrm{K} = \begin{pmatrix} -0.075 & 0 & 0 \\ 0 & -0.075 & 0 \\ 0 & 0 & -0.075s \end{pmatrix}$$

To obtain tyre forces without any change to the steering angle (which is set according to the Ackerman steering angle), a matrix B_f is defined as the linearised input matrix B with the terms related to the steering angle rate $\dot{\delta}$ zeroed out.

$$\mathrm{B_f} = \begin{pmatrix} I_{4\times4} & 0 \\ 0 & 0 \end{pmatrix} \times \mathrm{B}$$

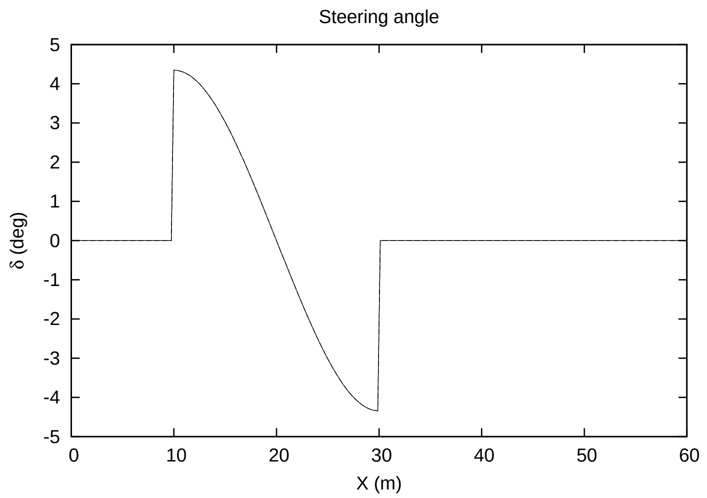
The necessary tyre force rates to produce the desired acceleration rate can then be obtained by inverting the linearised model

$$\dot{\mathbf{f}}_{\mathbf{x}} = \mathrm{B_f}^{-1}(\dot{\mathbf{w}} - \mathrm{A}\mathbf{w})$$

Note that $\mathrm{B_f}^{-1}$ here represents a pseudo-inversion using singular value decomposition; a true inversion is not possible because the matrix is singular. Appending $\dot{\delta}$, which is the steering angle rate required to produce Ackerman steering, to the force rate vector leads to the input rate vector

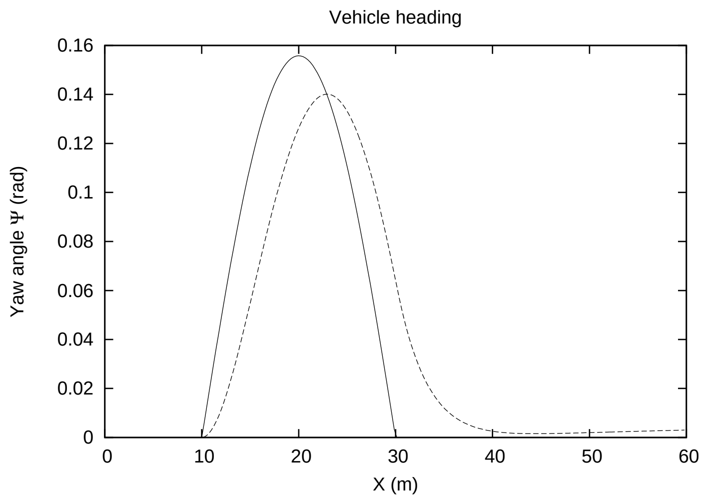
$$\dot{\mathbf{u}} = \begin{pmatrix} \dot{\mathbf{f}}_{\mathbf{x}} \\ \dot{\delta} \end{pmatrix}$$

which can be integrated to produce the final control input.

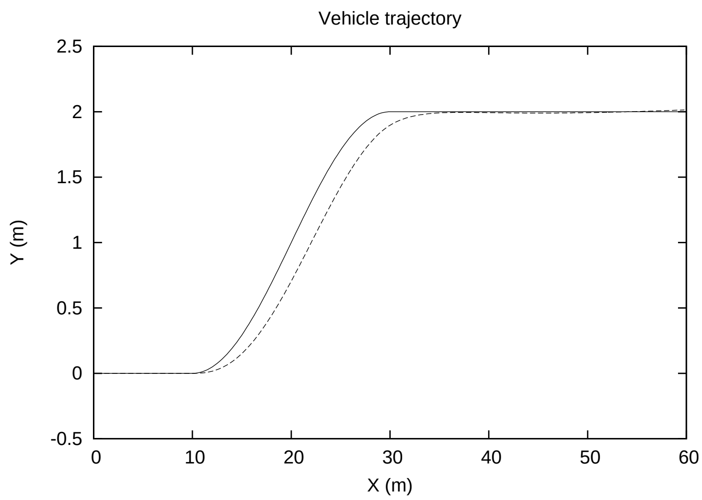


(a) Desired and actual steering angle during a lane-change manoeuvre. The steering wheel angle is set to the Ackerman angle for steady state cornering.

Figure 6: Simulation of a lane-change manoeuvre using steering and tyre force feedback control. The steering angle is set to the Ackerman steering angle required for steady state cornering. The additional feedback control allows the tyre forces to correct for deviation from the reference trajectory. The reference value is depicted by a solid line, the actual value by a dashed line.



(b) Desired and actual vehicle yaw angle. Wheel torques are applied through a feedback controller to correct the vehicle position and yaw rate.



(c) Desired and actual vehicle lateral position. Wheel torques are applied through a feedback controller to correct the vehicle position and yaw rate.

4 Case study: Yaw Stabilization of Automotive Vehicle Using Dynamic Control Allocation

4.1 Introduction

This section presents a dynamic control allocation scheme for lateral stabilization of an automotive vehicle. This theory development is related to Work-package 1 of the project, concerning active safety systems such as ESP, collision avoidance and rollover protection.

The algorithm takes as input the virtual control, which is the generalized forces commanded by some controller. This may be the moment around the yaw axis in case of an ESP or collision avoidance controller, the moment about the roll axis in a rollover protection system, or both moments in an integration dynamic vehicle management system. The output of the algorithm is the commands to a redundant set of actuators, such as the four individual brake pressures, and front wheel steering angle. The algorithm is based on a cost function and actuator constraints. The task is then to define an update law for the actuators that both searches to minimizes the cost subject to the constraints, and generates a real trajectory equal to the desired trajectory. The algorithm contains an adaptive mechanism to account to unknown parameters in the model of the generated forces, such as the tyre-road friction coefficient.

The algorithm is implemented and simulated using the DaimlerChrysler CASCaDE simulator.

4.2 The vehicle model.

The control design is based on an horizontal plane motion model found in (Ludemann 2002), it has the structure

$$\begin{aligned} \dot{x} &= f_1(t,x) + f_2(t,x)\tau \\ \tau &= h(t,x,u,\delta,\mu_{H(1..4)}) \end{aligned}$$

where $x := \left(\nu, \beta, \dot{\psi}\right)^{\mathrm{T}}$, $\tau := (f_x, f_y, M)^{\mathrm{T}}$ and $u := (\lambda_{x1}, \lambda_{x2}, \lambda_3, \lambda_{x4}, \Delta\delta)^{\mathrm{T}}$. The explicit model of the dynamic part is

$$\begin{aligned} f_1(t,x) &= f_1(x) := \left(0, -\dot{\psi}, 0\right)^{\mathrm{T}} \\ f_2(t,x) &: = \begin{pmatrix} \frac{1}{m}\cos(\beta) & \frac{1}{m}\sin(\beta) & 0 \\ -\frac{1}{m\nu}\sin(\beta) & \frac{1}{m\nu}\cos(\beta) & 0 \\ 0 & 0 & \frac{1}{J_z} \end{pmatrix} \end{aligned}$$

while the static control allocation model takes the following form

$$
\begin{aligned}
h(t,x,u,\delta_i,\mu_{H(1..4)}) &= h(x,u,\delta_i,\mu_{H(1..4)}) := \begin{pmatrix} \sum_{i=1}^{4} D(\delta_i,\Delta\delta)\begin{pmatrix} F_{xi} \\ F_{yi} \end{pmatrix} \\ \sum_{i=1}^{4} g^T(l_i,\theta_i) D(\delta_i,\Delta\delta)\begin{pmatrix} F_{xi} \\ F_{yi} \end{pmatrix} \end{pmatrix} \\
\begin{pmatrix} F_{xi} \\ F_{yi} \end{pmatrix} &: = \begin{pmatrix} -F_{zi}\mu_{xi}(\lambda_{xi},\alpha_{xi},\mu_H) \\ F_{zi}\mu_{yi}(\lambda_{xi},\alpha_{xi},\mu_H) \end{pmatrix} \\
D(\delta_i,\Delta\delta) &: = \begin{pmatrix} \cos(\delta_i+\Delta\delta) & -\sin(\delta_i+\Delta\delta) \\ \sin(\delta_i+\Delta\delta) & \cos(\delta_i+\Delta\delta) \end{pmatrix} \\
\lambda_{xi} &: = \frac{\nu-\omega_i r}{\nu}
\end{aligned}
$$

See Figure 7 and Table 1 for parameter explanation.

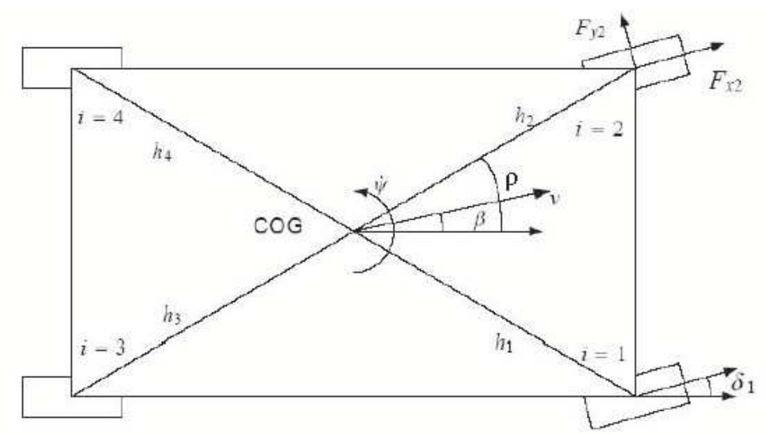


Figure 7: Horizontal plane car motion model.

4.3 Control Objective

The control objective is to stay within the following constraints

$$
\left|\dot\psi\right| \le \dot\psi_{\max} := \frac{a_{Y_{\max}} - \dot\nu\sin(\beta)}{\nu\cos(\beta)} + C \tag{1}
$$

$$
|\beta| \le 10^\circ - 7^\circ \frac{\nu^2}{(40m/s)^2} \tag{2}
$$

It is assumed that if (1) is satisfied, then (2) will be satisfied.

In Figure 8 a simulation example of a vehicle without an active yaw stabilization algorithm ($u = (0,0,0,0,0)^{\mathrm{T}}$) is shown.

4.4 Virtual control law

The high-level control design (the desired trajectory construction) is based on the reduced model (3) (Reduced by assuming that $\nu(t)$ and $\beta(t)$ bounds continuos time-

Nomenclature	
ν	Absolute velocity at the COG of the vehicle
β	Vehicle side slip angle
$\dot{\psi}$	Yaw rate
F_{xi}	Friction force on wheel in longitudinal wheel direction
F_{yi}	Friction force on wheel in lateral wheel direction
F_{zi}	Vertical force on ground from each wheel
δ_i	Steering angle
$\Delta\delta$	Commanded steering angle modification
M	Total yaw moment
M_u	Moment about COG due to the tyre slip
M_δ	Moment about COG due to steering angle
m	Vehicle mass
J_z	Vehicle moment of inertia about COG
μ_H	Maximum tyre road friction coefficient
α_i	wheel side slip angle
μ_{yi}	Lateral friction coefficient
μ_{xi}	Longitudinal friction coefficient

Table 1: Nomenclature

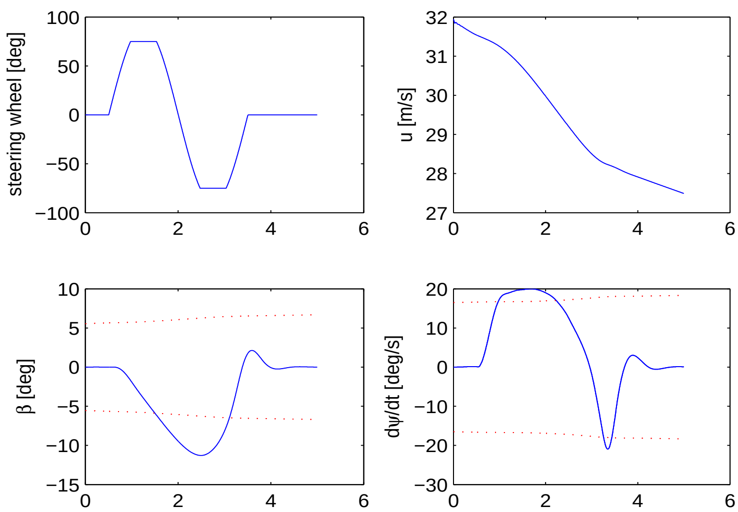


Figure 8: Absolut velocity, sideslip and yaw rate, for a vehicle whitout active braking.

varying signals)

$$\begin{aligned}\ddot{\psi} &= \frac{M}{J_z} = \frac{1}{J_z}\left(M_\delta + M_u\right) \quad (3)\\ M_\delta &: = M(t, 0, \delta, \mu_{H(1..4)})\\ M_u &: = M(t, u, \delta, \mu_{H(1..4)}) - M(t, 0, \delta, \mu_{H(1..4)})\end{aligned}$$

Define the coordinate transformation $\eta := \dot{\psi} - \dot{\psi}_{ref}$, where $\dot{\psi}_{ref}$ is assumed to be a constant, and let $M_u = -k\eta$ be the virtual controller, then by simple Lyapunov analysis we get

$$\begin{aligned}V_0(\eta) &= \frac{\eta^2}{2}\\ \dot{V}_0 &= \eta\dot{\eta}\\ &= \frac{1}{J_z}\left(-k\eta^2 + \eta M_\delta\right)\end{aligned}$$

such that the equilibria $\eta_{eq} = 0$ is Uniformly Globally Ultimately Bounded with ultimate bound $\frac{M_{\delta\max}}{k}$.

Note: *Since this bound can be made infinitesimal by the choice of arbitrary large k, the equilibria is said to be Uniform Global Practical Asymptotically Stable ($UGPAS$).*

As in (Tøndel and Johansen 2005*b*) we use the virtual control law defined by

$$M_{du} = M_u := \begin{cases} 0 & \left|\dot{\psi}\right| \leq \dot{\psi}_{\max} - C \\ -k\eta & else \end{cases} \quad (4)$$

where $\dot{\psi}_{ref} := sign(\dot{\psi})(\dot{\psi}_{\max} - C)$. It is here expected that this controller will render η_{eq}, $UGPAS$ by assuming that $\dot{\psi}_{ref}$ is slowly varying.

4.5 The optimization problem.

The static optimization problem is defined as

$$\min_u J(u) \qquad \text{s.t.} \quad M_{du}(t) - M_u(t, u) = 0$$

where $M_{du}(t, \dot{\psi})$ is our desired virtual control (desired trajectory) and $M_u(t, u)$ is what actually is preformed by manipulating the actuator vector u. The cost function $J(u) = J_1(u) + J_2(u)$ is divided into two parts. J_1 represents the actuator cost and J_2 is a barrier function representation of the actuator constraints.

$$\begin{aligned}J_1 &= a^2 u^{\mathrm{T}} \Gamma_u u\\ J_2 &= -w_1 \sum_{i=1}^{4} \ln(\lambda_{xi} - \lambda_{x\min}) - w_1 \sum_{i=1}^{4} \ln(-\lambda_{xi} + \lambda_{x\max})\\ &\quad -w_1 \ln(\Delta\delta - \Delta_{\min}\delta) - w_1 \ln(-\Delta\delta + \Delta_{\max}\delta)\end{aligned}$$

The Lagrangian function is then given by

$$l(t, u, \lambda) = J(u) + (M_{du}(t) - M_u(t, u))^{\mathrm{T}}\lambda \quad (5)$$

4.6 Dynamic control allocation algorithms

4.6.1 Motivation.

- Construct update laws $\dot{u}$ and $\dot{\lambda}$ such that some attraction and stability properties of the set $\mathcal{O} := \{z \in \mathbb{R}^q \, | G(z) = 0\}$, where

$$\begin{aligned} z &: = (p, \eta, u, \lambda)^{\mathrm{T}} \\ \dot{p} &= 1 \\ G(z) &: = \left(\eta, \frac{\partial l}{\partial u}, \frac{\partial l}{\partial \lambda}\right)^{\mathrm{T}} \end{aligned}$$

can be proved. (see (Tjønnås and Johansen 2005) and fix θ, for details)

Note: *The first order optimal condition is satisfied by* $\frac{\partial l}{\partial u} = 0, \frac{\partial l}{\partial \lambda} = 0$.

4.6.2 Direct Lyapunov based design (μ_{Hi} known and equal for all tyres).

Consider the following Lyapunov function candidate

$$V_1(t, \eta, u, \lambda) := \sigma V_0 + \frac{1}{2}\left(\frac{\partial l^{\mathrm{T}}}{\partial u}\frac{\partial l}{\partial u} + b^2 \frac{\partial l^{\mathrm{T}}}{\partial \lambda}\frac{\partial l}{\partial \lambda}\right) \tag{6}$$

Then by applying a update-law of the same from as in (Johansen 2004)

$$\dot{u} = -\Gamma(t)\alpha_u + \zeta \tag{7}$$

$$\dot{\lambda} = -W(t)\beta_u + \phi \tag{8}$$

$$\alpha_u^{\mathrm{T}}\zeta + \beta_u^{\mathrm{T}}\phi + \delta_u = 0 \tag{9}$$

where α_u, β_u and δ_u (not related to the geometrical parameters of the vehicle) are functions dependent on the Lagrangian partial derivatives, t and η, it can be shown that

$$\dot{V}_1 \leq -c_1 \|\eta\|^2 - c_2 \left\|\frac{\partial l}{\partial u}\right\|^2 - c_3 \left\|\frac{\partial l}{\partial \lambda}\right\|^2 \qquad \forall \|\eta\| > \vartheta \tag{10}$$

Further more ϑ will be reduced for growing k such that $UPAS$ (Not $UGPAS$, since the actuators are constrained) may be concluded for the set $\mathcal{O}$. The results from this section are presented in Figure 9 and 10. For all simulations, the time scales are given in seconds and the sampling rate is $5ms$.

4.6.3 Assume μ_{Hi} unknown.

Note: *In the adaptive case studies, we only consider the case where the model structure is exactly known. Thus we simulate the control laws directly on the model which the laws was based upon, in stead of the CASCaDE model. An implementation in CASCaDE will be presented later, but to date there are some sensitivity issues in the implementation that needs to be solved.*

As before we consider the model

$$\ddot{\psi} = \frac{1}{J_z}\left(M_\delta(t, 0, \delta, \mu_{H(1..4)}) + M_u(t, u, \delta, \mu_{H(1..4)})\right)$$

But since the unknown parameter μ_{Hi} is not affine in this model the theory from (Tjønnås and Johansen 2005) can not be used directly.

Consider a Taylor approximation by linearization around $\hat{\theta} = \hat{\mu}_H$, such that

$$\begin{aligned}
M(..,\theta) &= M(..,\hat{\theta}+\tilde{\theta}) \\
&= M(..,\hat{\theta}) + \frac{\partial M}{\partial \hat{\theta}}\tilde{\theta} + err \\
M(..,\theta) - M(..,\hat{\theta}) &= \frac{\partial M}{\partial \hat{\theta}}\tilde{\theta} + err \qquad (11)
\end{aligned}$$

thus we can use the estimation model

$$\begin{aligned}
\ddot{\hat{\psi}} &= A\epsilon + \frac{1}{J_z}\left(M_\delta(t,0,\delta,\hat{\theta}) + \hat{M}_u\right) \qquad (12) \\
\hat{M}_u &= \left(M(t,u,\delta,\hat{\theta}) - M(t,0,\delta,\hat{\theta})\right) \\
\epsilon &= \dot{\psi} - \dot{\hat{\psi}} \qquad (13)
\end{aligned}$$

By the parameter update-law

$$\dot{\hat{\theta}} = Q_\theta^{-1}\left(\Phi_\theta^{\mathrm{T}}\Gamma_\theta^{\mathrm{T}} + \epsilon^{\mathrm{T}} Q_\epsilon \frac{1}{J_z}\frac{\partial M(t,0,\delta,\hat{\theta})}{\partial \hat{\theta}}\right) \qquad (14)$$

where $\Phi_\theta := \frac{1}{J_z}$ and Γ_θ are functions of the Lagrangian partial derivatives, t, η, ϵ and $\hat{\theta}$, and the certainty equivalent controller from (7-9), the Lyapunov function candidate

$$\begin{aligned}
V_2(t,\eta,\epsilon,\hat{\theta},u,\lambda) := \sigma V_0(t,\eta) + \frac{1}{2}\tilde{\theta}^{\mathrm{T}} Q_\theta \tilde{\theta} + \frac{1}{2}\epsilon^{\mathrm{T}} Q_\epsilon \epsilon \\
+ \frac{1}{2}\left(\frac{\partial l^{\mathrm{T}}}{\partial u}\frac{\partial l}{\partial u} + b^2 \frac{\partial l^{\mathrm{T}}}{\partial \lambda}\frac{\partial l}{\partial \lambda}\right) \qquad (15)
\end{aligned}$$

has the following property

$$\dot{V}_2 \le -c_{12}\left\|\eta\right\|^2 - c_{22}\left\|\frac{\partial l}{\partial u}\right\|^2 - c_{32}\left\|\frac{\partial l}{\partial \lambda}\right\|^2 - c_{42}\left\|\epsilon\right\|^2 \qquad \forall \left\|\eta, \frac{\partial l}{\partial u}, \frac{\partial l}{\partial \lambda}, \epsilon\right\| > \vartheta_2 \quad (16)$$

It can be shown that $\frac{\partial l}{\partial u}, \frac{\partial l}{\partial \lambda}, \epsilon \to 0$ as $t \to \infty$ and η converges to some bound dependent on k. If in addition the signal $\frac{\partial M}{\partial \hat{\theta}}(t)$ is Persistently exited (PE), then $UPAS$ can be proved.

4.7 Implementation and simulation results

Four simulation scenarios are presented, all based on a *Mercedes S* class model, the steering manoeuvre and the initial conditions shown in Figure 8. The first two scenarios are simulated on the CASCaDE model while the last two are adaptive cases simulated on the ideal model presented. The control update-laws are as in (Johansen 2004) defined by

$$\left(\dot{u}, \dot{\lambda}\right)^{\mathrm{T}} = -\gamma\left(\mathbb{H}^{\mathrm{T}}\mathbb{H} + \varepsilon I_{r+p}\right)^{-1}\left(\alpha_u, \beta_u\right)^{\mathrm{T}} + (\zeta, \phi)^{\mathrm{T}} \qquad (17)$$

with the difference that γ is a positive definite diagonal matrix instead of a constant. The remaining coefficients are given by

$$\begin{aligned}
\mathbb{H} \quad &: = \begin{pmatrix} \frac{\partial^2 \ell}{\partial u^2} & -(\frac{\partial M_u}{\partial u})^{\mathrm{T}} \\ -(\frac{\partial M_u}{\partial u}) & 0 \end{pmatrix} \\
\alpha_u \quad &: = \frac{\partial^2 l}{\partial u^2}\frac{\partial l}{\partial u} - b^2 \frac{\partial M_u^{\mathrm{T}}}{\partial u}\frac{\partial l}{\partial \lambda} \\
\beta_u \quad &: = -\frac{\partial M_u}{\partial u}\frac{\partial l}{\partial u} \\
\delta_u \quad &: = (\frac{\partial l^{\mathrm{T}}}{\partial u}\frac{\partial^2 l}{\partial \eta \partial u} + b^2 \frac{\partial l^{\mathrm{T}}}{\partial \lambda}\frac{\partial^2 l}{\partial \eta \partial \lambda})M
\end{aligned}$$

For the adaptive case we use the parameter update law from (14) with the following adaptive gain

$$\Gamma_\theta = \left(\frac{\partial l^{\mathrm{T}}}{\partial u}\frac{\partial^2 l}{\partial \eta \partial u} + \frac{\partial l^{\mathrm{T}}}{\partial \lambda}\frac{\partial^2 l}{\partial x \partial \lambda} + \epsilon^{\mathrm{T}} Q_\epsilon + \sigma \frac{\partial V_0^{\mathrm{T}}}{\partial \eta} \right)$$

and use the certainty equivalent algorithm from (17). The algorithm parameters used in the simulations are given by

C	$5\frac{\circ}{s}$	Q_θ	$10^2 b^2 a^2$
$a_{Y_{\max}}$	0.8	Q_ϵ	$10^3 Q_\theta$
ε	10^{-8}	A	10^2
a	10^6	σ	1
Γ_u	$diag(1,1,2,2)$	$\lambda_{x\min}$	-10^{-5}
b	130	$\lambda_{x\max}$	$0.2 + 10^{-5}$
k	$20 J_z$	$\Delta_{\min}\delta$	$-25°$
c	a	$\Delta_{\max}\delta$	$25°$
w_1	1		

4.7.1 Friction model modification

In Figure 9 the real and model based moments around COF of the vehicle is recorded, from a simulation without applying any brake force. Due to this trajectory errors, the friction model is modified by multiplying λ_{xi} with 1.2 and λ_{yi} with 0.7 for each wheel. The result of this change is shown in the lower plot in Figure 9. The real trajectory is calculated by the measured forces on each wheel.

4.7.2 Simulation results

In the nonadaptive cases we assume that the friction between the tyres and the road is known and equal for all wheels ($\mu_H = 0.8$). The CASCaDE model control inputs are defined through the steering angle and an ABS block where desired brake force for each wheel in the longitudinal direction can be specified. The desired forces are estimated by inserting the solution of the control allocation problem in the friction model from the model section.

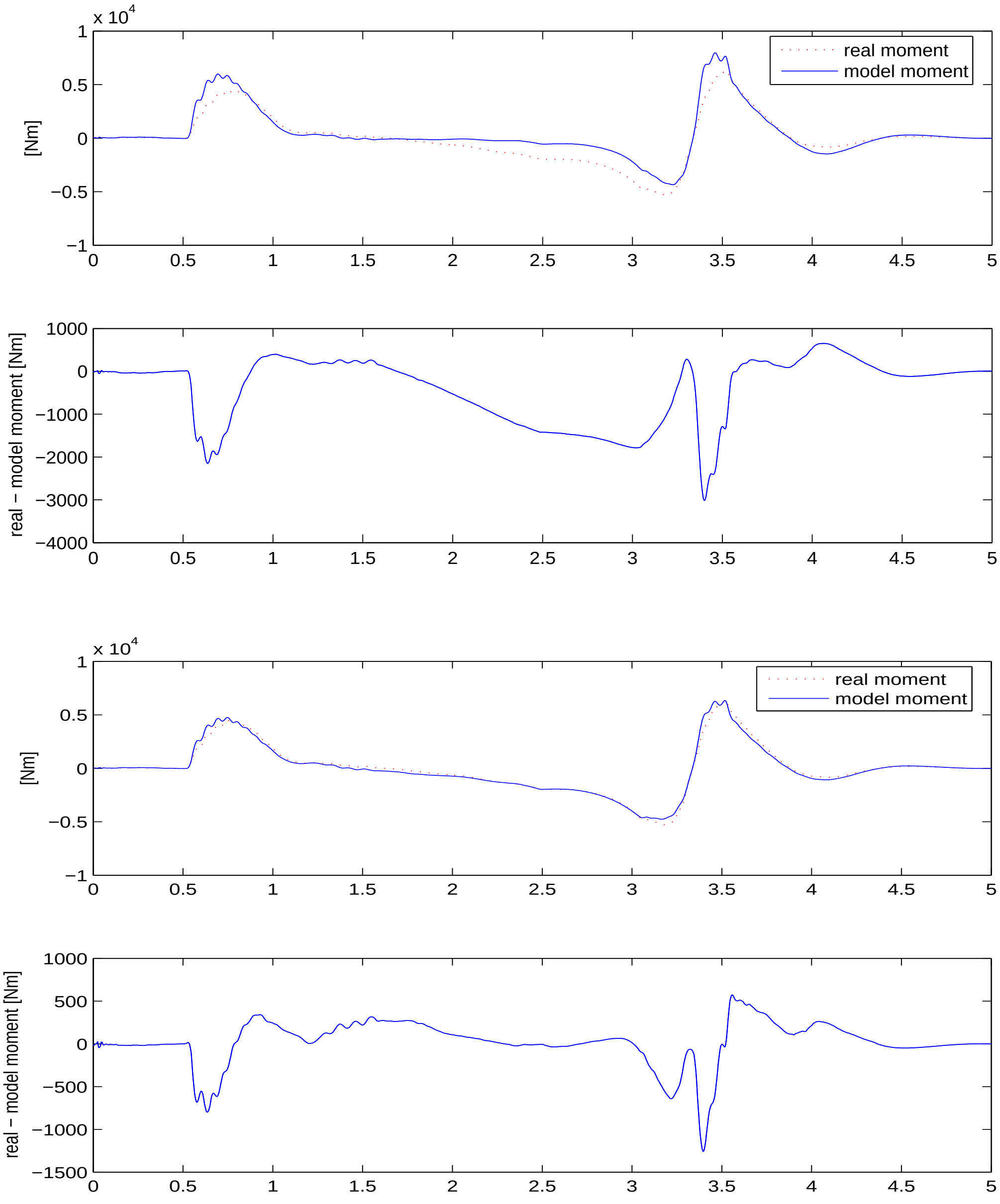


Figure 9: Model comparison. The real moment is compared with the initial model (on top) and a modified model (bottom)

In the first scenario we fix $\Delta\delta = 0$ and consider two ways of generating the desired longitudinal wheel slip. In the first case (Figure 10), u is given by the CASCaDE model and used directly in the update algorithm for longitudinal wheel slip. In the second case u is considered to be a virtual desired state, defined by the updated law (no direct feedback of u from the CASCaDE model). The simulation results of this indirect update are presented in Figure 11.

In both of the update schemes, the initial control objectives are met.

In the second scenario $\Delta\delta$ is also part of the allocation problem, the results are shown in Figure 12.

From the trajectories shown in Figure 10 and 11 it can be seen that the direct update approach generates a "real" moment (calculated based on the model presented) that closer to the desired moment. By introducing an extra actuator the tracking performance is further improved (comparing the trajectories in Figure 11 and 12). The advance in performance from indirect to direct and an additional actuator, fits the intuitive understanding of the allocation problem.

The model error is both apparent in the update law and in the mapping from desired longitudinal slip to desired longitudinal brake forces. Due to this model error, the adaptive update law not simulated on the CASCaDE model with success. Model improvement and robustification schemes are currantly the main focus, in the work of introducing the adaptive algorithm.

The performance of the adaptive update laws are shown by considering an error estimator model with the same structure as the real model. In our first adaptive scenario we assume that μ_{Hi} is unknown, but equal for all tyres on the vehicle with the initial parameter guess

$$\begin{aligned} \hat{\mu}_{H(1..4)init} &= 0.2 \\ \mu_{H(1..4)} &= 0.8 \end{aligned}$$

The simulation results are shown in Figure 13.

In the second plot we assume that μ_{Hi} unknown, but equal for tyres on the same side of the vehicle. This will be the case for a car that is partially off the road. The same certainty equivalent control algorithm as before is used, but with an adaptive law that estimates two parameters. The results are shown in Figure 14.

$$\begin{aligned} \hat{\mu}_{H(left)init} &= 0.4 \\ \hat{\mu}_{H(right)init} &= 0.3 \\ \mu_{H(left)} &= 0.6 \\ \mu_{H(right)} &= 0.8 \end{aligned}$$

The main control objective on the sideslip angle is satisfied in both the adaptive cases, but the control objective related to the yaw rate is violated at peaks between 2 and 3 seconds.

4.8 Further work

The implementation of an optimizing nonlinear control allocation algorithm on an automotive vehicle was presented in this section. Longitudinal slip for each wheel

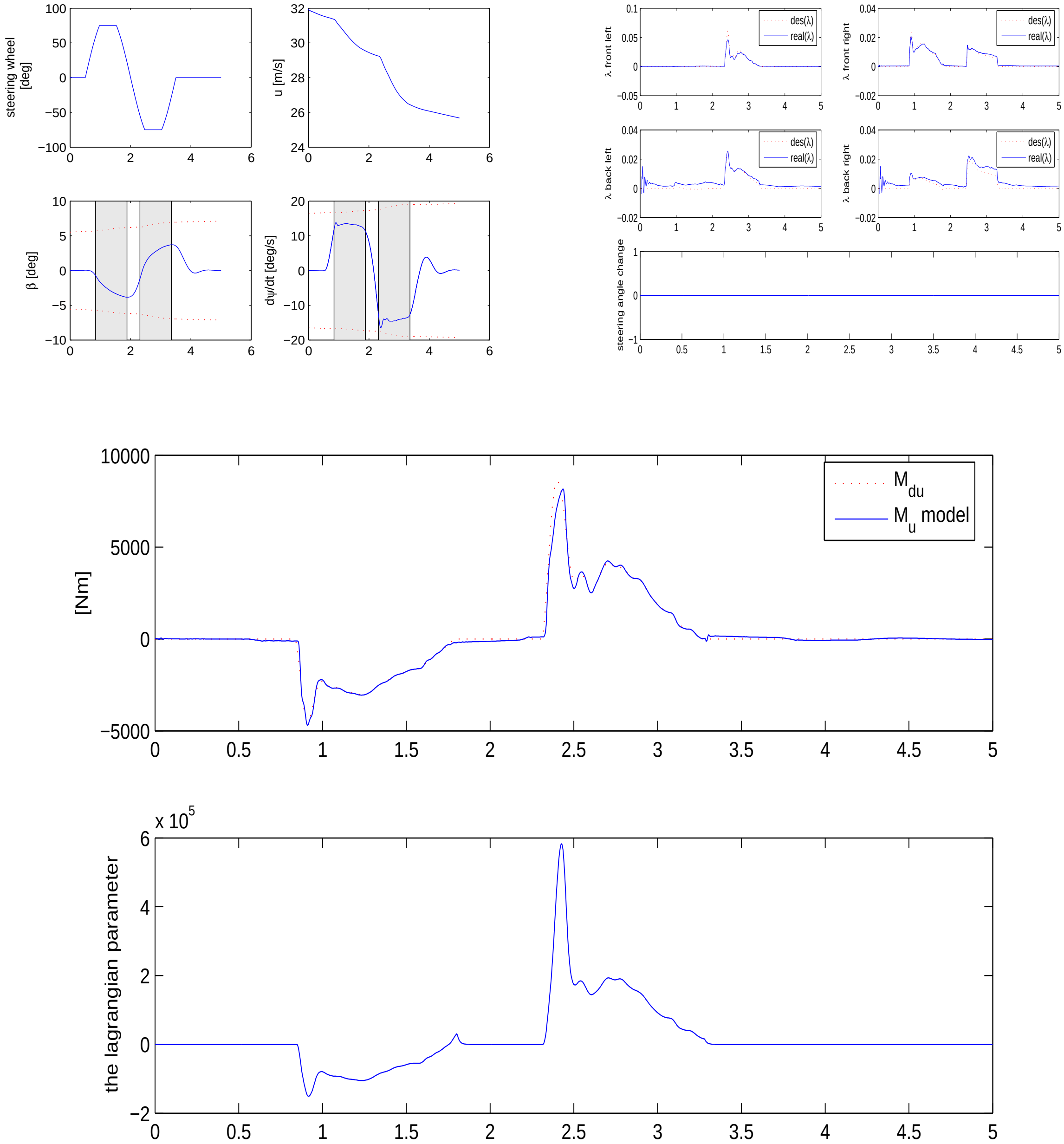


Figure 10: Simulation results where the real slip is directly updated. The shaded area marks the timeinterval of feedback control.

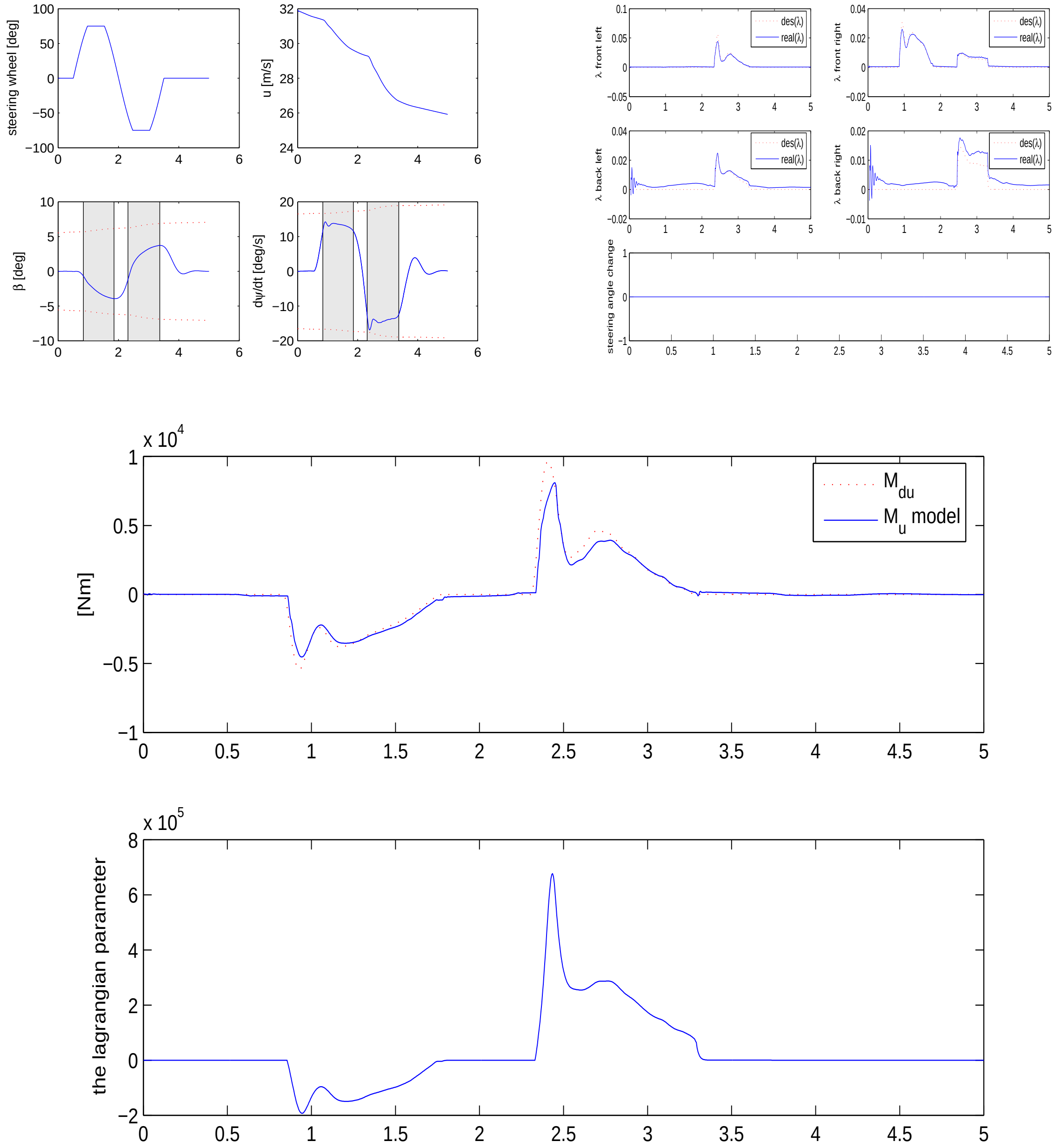


Figure 11: Simulation results where the control moldel slip is indirectly updated. The shaded area marks the timeinterval of feedback control

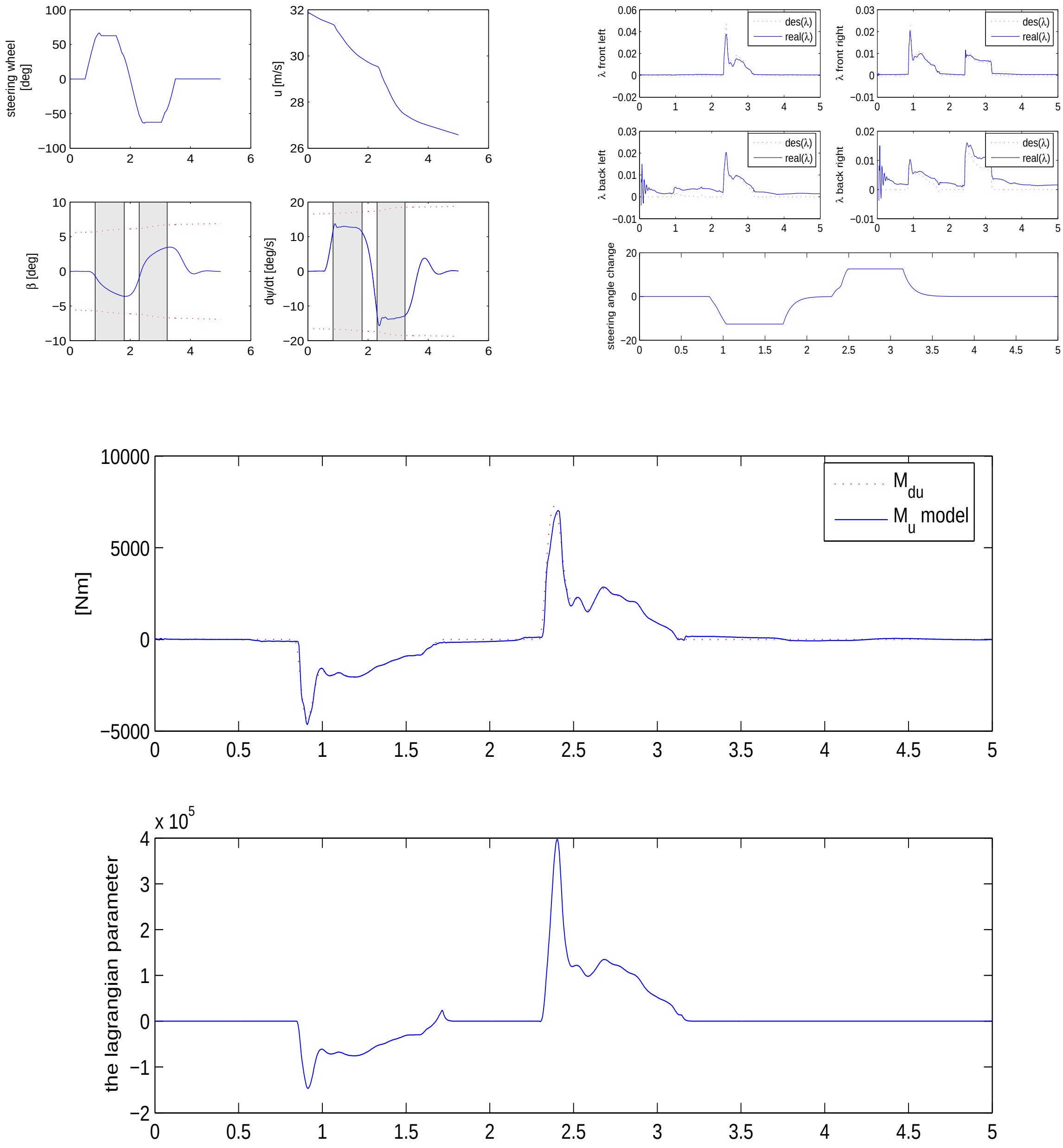


Figure 12: Simulation results where the slip and steering angle difflection are directly updated. The shaded area marks the timeinterval of feedback control

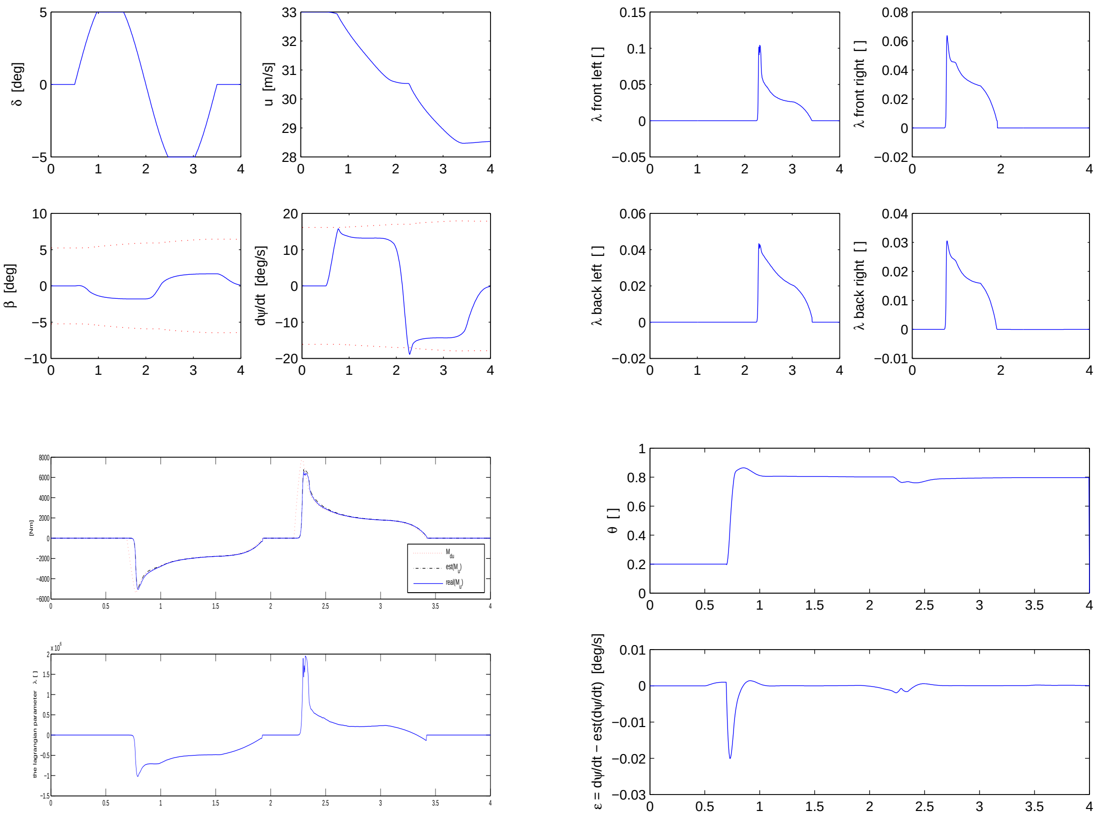


Figure 13: Unknown but equal frictionparameters

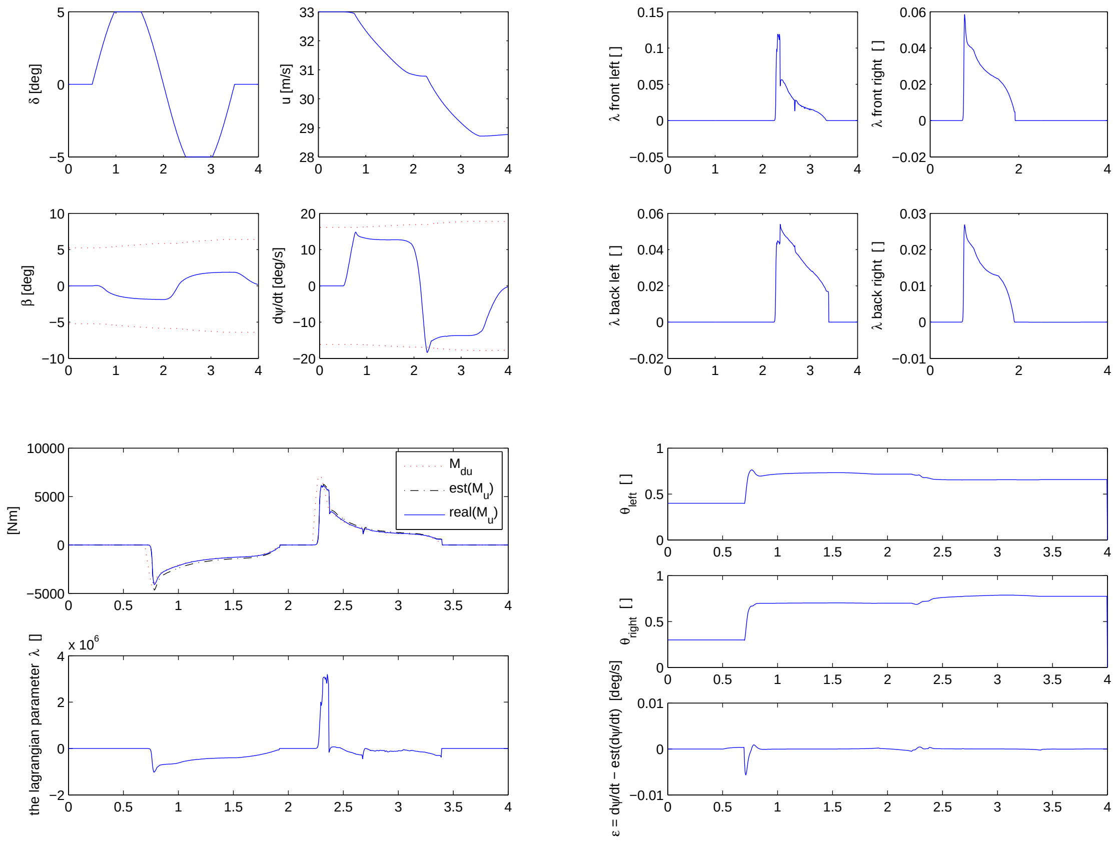


Figure 14: Unknown but equal friction parameters for tires on the same side of the car

was considered as actuators for yaw rate stabilization of the automotive vehicle, and simulations were carried out on the Mercedes S class model from (Ludemann 2002) and CASCaDE (nonadaptive case)

Further work will include the following topics

- Implement an adaptive law for the tyre road friction parameter on the CASCaDE simulator.
- Investigate the possibility of using active suspentions in attidude control and manipulate the steering angle indirectly in stead of directly as shown here.
- Design an integrated dynamic vehicle management system for both roll-over prevention and collision avoidance.

Theoretical work on combining set stability with cascaded system theory will be carried out to simplify both the analysis and the allocation design. Simulation results are promising especially for the adaptive case, where the modularity introduces the possibility to use a wide range of relatively simple adaptive update laws.

5 Summary and Conclusions

In this report, an overview was given on the development of an integrated set of control development tools and design strategies which will be used in the application work packages. For each application, which include roll-over protection (WP 1.1), collision avoidance (WP 1.2) and integrated chassis control (WP 2), a set of appropriate design methods was identified and tools were described.

Design methodolgies cover all parts of work package 3, ranging from classical multivariable design in the form of ICAD, over hybrid control systems where multiple-control systems and control allocation methods were introduced, to systems with time-varying time delays and nonlinear and adaptive control. It was shown how the theoretical developments in work package 3 are motivated and driven by the requirements of the applications where the design tools are to be applied.

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