Complex Embedded Automotive Control Systems
CEmACS

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State of the Art of Integrated Chassis Control
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Executive Summary

The development of Integrated Chassis Control (ICC) systems is a major area of current interest in automotive research. Roughly speaking, research on this topic has been motivated by the desire for increased vehicle safety, increased comfort, and better performance by taking into account and utilising the interactions that exists between the different subsystems of the vehicle. While research on this topic has progressed along several lines of inquiry, it appears to be the case that almost no effort has been devoted toward the construction of vehicle emulators that are based on ICC concepts. For example, vehicles equipped with active systems (4-wheel-steering, active suspension, active brakes, etc.) may, in principle, be constructed to emulate any reasonable given set of vehicle dynamics. Arguments for the design of such vehicle emulators are compelling. In particular, vehicle designers should in principle be able to use vehicle emulators to test prototypes before their construction and at a lower expense than by using vehicle simulators and actual prototype vehicles.

The objective of this work package (WP2) is to develop an ICC with the capacity to emulate a vehicle with a given set of vertical and lateral dynamics using the vehicle suspension and steering systems (and taking into account the interaction between these systems). As a first step in this direction we have reviewed work that has been carried out in ICC; with particular emphasis on work that integrates both suspension and steering subsystems. In this report, a brief overview of this work, together with a review of suspension and steering control systems and their interactions is presented. A 4-wheel-steering controller developed at the Hamilton Institute as part of a CEMACS pre-project is also described. Finally, we present some preliminary research results; in particular, a vehicle model that integrates the vehicle steering and suspension systems.

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Contents

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1 Introduction

The concept of generic prototype vehicles has emerged as a promising solution to an outstanding challenge in the development of ride and handling characteristics for advanced passenger cars: the bridging of the gap between numerical simulations based on a vehicle model—a virtual prototype—and experiments on a proof-of-concept prototype vehicle. A generic prototype vehicle would be equipped with advanced computer-controlled actuators enabling it to modify its ride and handling characteristics. Examples of such advanced actuators are four and rear steer-by-wire, brake-by-wire and active suspensions. An integrated chassis controller would command those actuators to track a set of reference signals corresponding to a desired ride and handling behaviour. Currently, moving-base driving simulators are used to emulate the ride and handling behaviour of virtual prototypes prior to building real ones. However, the achievable accelerations of such simulators severely constrain their ability to realistically recreate the full range of vehicle motion. Generic prototype vehicles could allow for the realistic recreation of the ride and handling characteristics of virtual prototypes, thereby enabling engineers to experience and evaluate their behaviour prior to making the decision of building expensive proof-of-concept prototypes.

The objective of this work package (WP2) is to develop an ICC with the capacity to emulate a vehicle with a given set of vertical and lateral dynamics using the vehicle suspension and steering systems (and taking into account the interaction between these systems). As a first step in this direction we have reviewed work that has been carried out in ICC; with particular emphasis on work that integrates both suspension and steering subsystems. In this report, a brief overview of this work, together with a review of suspension and steering control systems and their interactions is presented. A 4-wheel-steering controller developed at the Hamilton Institute as part of a CEMACS pre-project is also described. Finally, we present some preliminary research results; in particular, a vehicle model that integrates the vehicle steering and suspension systems

2 Control of lateral dynamics

We begin our discussion by considering the lateral motion of the vehicle. By lateral motion of the vehicle we mean how the vehicle responds to a steering input. A driver controls the vehicle lateral dynamics by indirectly affecting the forces generated by the tyres of the vehicle. Roughly speaking, these forces are affected by several vehicle systems; (i) the vehicle steering system; (ii) the vehicle braking susbsystem (especially with differential braking); (iii) the vehicle suspension system; and (iv) the vehicle drivetrain. Clearly, the response of the vehicle to a steering input is most affected by the steering system. However, the braking system, the suspension system, and the vehicle drivetrain may also be used to influence the steering capabilities of the vehicle; it is therefore not surprising that research on controlling the lateral motions of the vehicle has recently concentrated on integrating these systems.

2.1 Controlling vehicle lateral motion through vehicle steering systems

Traditionally, vehicle steering systems have been used to control the lateral motion of the vehicle. Roughly speaking, work in this area can be broken down along the following lines; work on active front steering; active rear steering; and 4 wheel steering systems.

The steering controller system may influence the direction of the tyres in different ways as depicted in Figure 1. Forward steering controllers alters the direction of front wheels as a function of the driver input with or without mechanical link. Rear steering controllers on the other hand do not influence the front steering angle (this task is left to the driver) but rather affect the vehicle dynamics by adjusting the steering angles of the rear wheels. 4-wheel-steering systems control both front and rear steering angles as a function of driver inputs and vehicle dynamics.

A front wheel steering system without mechanical link between the steering wheel and the wheels steering angle (i.e. a front steer-by-wire system) is presented in [1]. The control structure uses feedback and feedforward control as a function of the steering wheel input, the measured speed, yaw-rate and lateral acceleration measurements, to control the vehicle yaw-rate. The resulting controller is shown to have a better vehicle stability during a lane-change manoeuvre over packed snow than one with brake and drive force distribution through Direct Yaw-moment Control (DYC). In addition, the resulting system feeds back a torque signal to the driver via the steering wheel. Other proposals for active steering keep direct mechanical link from the steering wheel but add an additional steering angle through the use of an electrical motor mounted in the steering system. This kind of system can, in principle, allow steering in case of control system failures, and provides enhanced safety functionality. This kind of structure is used in [2] and [3]. In the paper by J.Ackermann [3], the driver task of lane keeping and the automated yaw stabilization are separated via yaw-rate feedback control system correcting the steering angle. The test results given in this paper show robustness to crosswind perturbations as well as to μ-split-braking. Another approach is presented in [4]. In this work a steer-by-wire system and a conventional steering system are integrated into a single steering system. This construction allows for the introduction of a safety management system that reverts to normal steering in case of failure of the steer-by-wire function.

Active rear steering has been studied by several authors for controlling vehicle lateral dynamics. Most of these structures rely on the use of gain-scheduled feedforward control to command the rear steering angle [5]. In such control structures, some of which have already been implemented on production passenger cars, the rear steering angle is computed as a function of the front steering angle command from the driver's input to the steering wheel. Several different control laws have been proposed for controlling such systems (usually related to the improvement of the manoeuvrability and cornering stability of the vehicle). Inoue et al.[6] and, some years later, Hirano et al. [7] combine feedforward and feedback control to command the rear steering angle, while the front steering angle remains under direct control

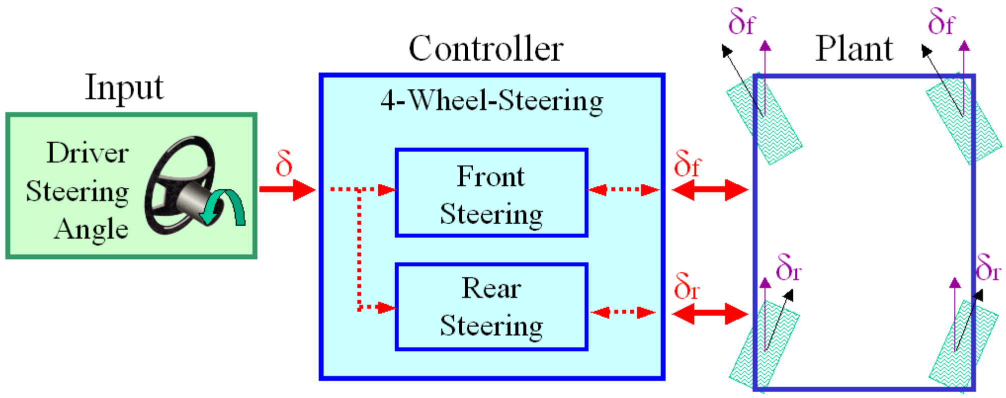


Figure 1: Active steering systems

of the driver. The control objective here is to follow a predefined model of the vehicle dynamics. In order to achieve a satisfactory degree of robustness the feedback controller is designed using μ synthesis by Hirano et al. The results of the latter demonstrate improved handling and stability in a variety of experimental conditions.

In some situations it is desirable to control both side-slip angle and the vehicle yaw-rate[8]. The control of both of these dynamics simultaneously is not possible using only active rear, or only active front, steering. The control of both of these signals requires at least two control inputs. In vehicles equipped with the Electronic Stability Program (ESP) or DYC, this may be achieved through the brake and drive force (one input) and by using the the front steering angle as a second input. Clearly, using ESP to control lateral dynamics is not desirable in all situations. For vehicles operating under normal operation conditions, controlling lateral dynamics using 4-wheel-steering is clearly desirable; here the front and rear steering angles are the two control inputs. In this case, even undesired side effects from ESP or DYC can be counteracted. The ability to control side-slip and yaw-rate independently of each other is depicted in Figure 2; when the front and rear steering angles are in opposite directions, a yaw-rate without sideslip can be achieved so a constant radii curve could be managed: when both angles have the same direction, a sideslip angle without yaw-rate can be performed to change lanes.

A substantial body of research on the control of 4-wheel steering cars already exists and a variety of control structures have been proposed in the literature. Recently, work at the Hamilton Institute has been carried out in this area[9]. In this paper, a new feedback steering controller capable of modifying the lateral dynamics of 4-wheel steering cars to follow a given reference model is presented. The proposed steering controller commands the front and rear steering angles with the objective of tracking reference sideslip angle and yaw rate signals obtained online from the driver's inputs to steering wheel and pedals. These reference signals describe the lateral motion that would result if those driver's inputs were applied to a vehicle model with the desired dynamics. In addition, the steering controller automatically rejects any disturbances in sideslip and yaw rate. The work described in [10] proposes a

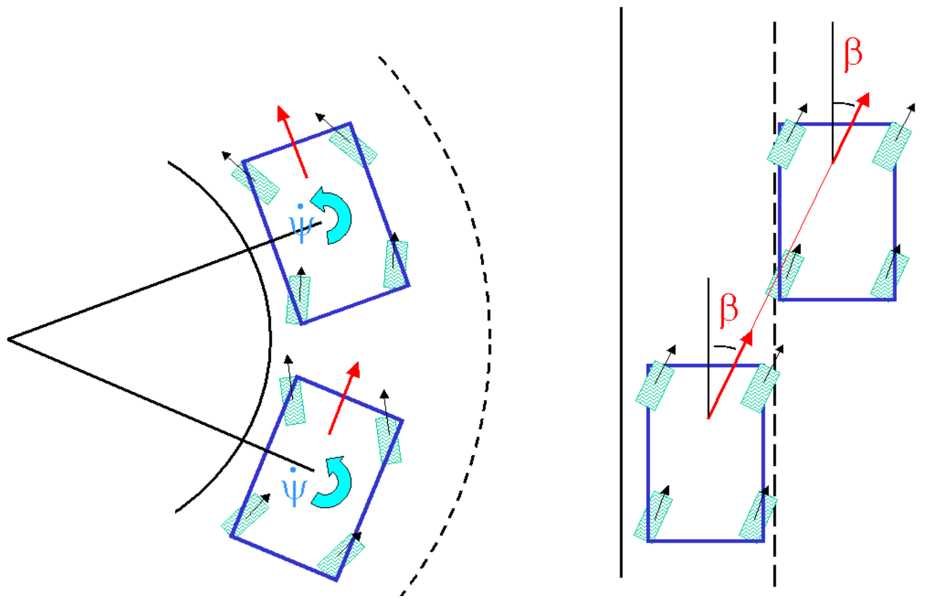


Figure 2: (a)Independent Yaw-rate and (b)Independent Side-slip

feedback control structure based on Virtual Model Following Control and robust LQR design. The model to be followed corresponds to the front-wheel steered car. An example of an steering controller specifically designed for cars equipped with 4-wheel steer-by-wire is presented in [11]. The controller structure given in this paper is based on the cross-feedback of the measured yaw rate to the front steering angle. This structure decouples the control of the lateral acceleration from the control of the yaw rate. Two outer feedback loops are used so that front wheel steering is used to track the desired lateral acceleration and rear wheel steering is used to regulate the damping of the resulting yaw dynamics.

In the bibliography that accompanies this report, few of the listed papers to the vehicle emulation problem and only in [12], results for an emulator where presented. In this paper[12], a "midsized" vehicle with an active rear steering algorithm is used to emulate lateral dynamics of "small", "compact" and "full-sized" vehicles. The emulation test results for "small" vehicles are good but not for "compact" and "full-sized" ones.

2.2 4-wheel steering controller developed at the Hamilton Institute

A new 4-wheel steering concept was developed at the Hamilton Institute as part of a CEMACS pre-project and is documented in [9]. The controller developed here aims to be part of the integrated chassis controller of a generic prototype test vehicle and the work developed by the author and his co-authors will be used in this project. The structure of the steering controller is based on a simplified linear model of the lateral dynamics of 4-wheel steering cars at constant speed. The main elements of the controller structure are a linear input transformation, a sideslip cross-feedback and a speed-dependent yaw rate feedback. The former two elements result in the partial decoupling of the sideslip and yaw rate responses with respect to the transformed inputs and the latter yields speed-invariant yaw rate dynamics, thereby acting as an implicit gain scheduling on the vehicle speed. The original 2-by-2 multivariable control design problem can then be recast as two SISO control design problems according to the ICD paradigm[13]. A more accurate model including the steering actuator dynamics as well as the communication time delay between controller and actuators is then considered. When applied to this new model, the proposed controller structure results in approximate partial decoupling and nearly speed-invariant yaw rate dynamics. Assuming certain bandwidth restrictions, controllers for the resulting sideslip and yaw rate channels can be designed individually within the proposed structure using SISO techniques. The resulting controllers satisfy diturbance rejection requirements. A feedforward element is introduced to improve the response to reference inputs. Finally, an anti-windup scheme is incorporated into the controller to mitigate the effects of saturation or failure of the rear steering actuators. The resulting non-linear steering controller is shown to be valid for varying vehicle speed and shows excellent robustness and integrity with respect to rear actuator saturation or failure. This controller was implemented in simulation and tested in an S Class MercedesBenz test vehicle. This work will be futher developed as part of the vehicle emulator by integrating the vertical dynamics of the vehicle.

In order to illustrate the performance of the controller currently available for lateral vehicle dynamics emulation, some results for simulation in μ-split braking situations as well as experimental maneuvers showing desired dynamics tracking performance are presented. In a μ-split braking situation the car brakes with the tyres at opposite sides of the vehicle on different local road conditions. This results in the tyres at one side of the car see an adhesion coefficient (μ) different from the one seen by the tyres at the other side. An example of μ-split braking is a car braking with the two wheels at one side over a patch of ice and the other two on dry asphalt. In μ-split braking the torque created by the difference between the braking forces at either side of the vehicle introduces disturbances in both yaw rate and sideslip. These disturbances may induce the car to spin and cause the driver to lose control of the vehicle. The proposed steering controller automatically rejects any disturbances in sideslip and yaw rate generated in a μ-split braking situation. To illustrate this capability, consider the following example. A car travels along a straight level road at a speed of 60 m/s. At some point, the driver starts braking without turning the steering wheel. Suppose that the two wheels at the left hand side of the car are on dry asphalt ($\mu \approx 1$) and the two on the right hand side are on ice ($\mu \approx 0.2$). Since the driver keeps the steering wheel straight, the reference signals to be tracked by the steering controller are zero rad/s yaw rate and zero rad sideslip. Figure 3 illustrates the result of simulating this manoeuvre with and without the steering controller switched on. It can be seen that without the steering controller the car spins. On the other hand, with the controller in place the disturbances are quickly rejected and the car barely deviates from its intended straight path. The performance of

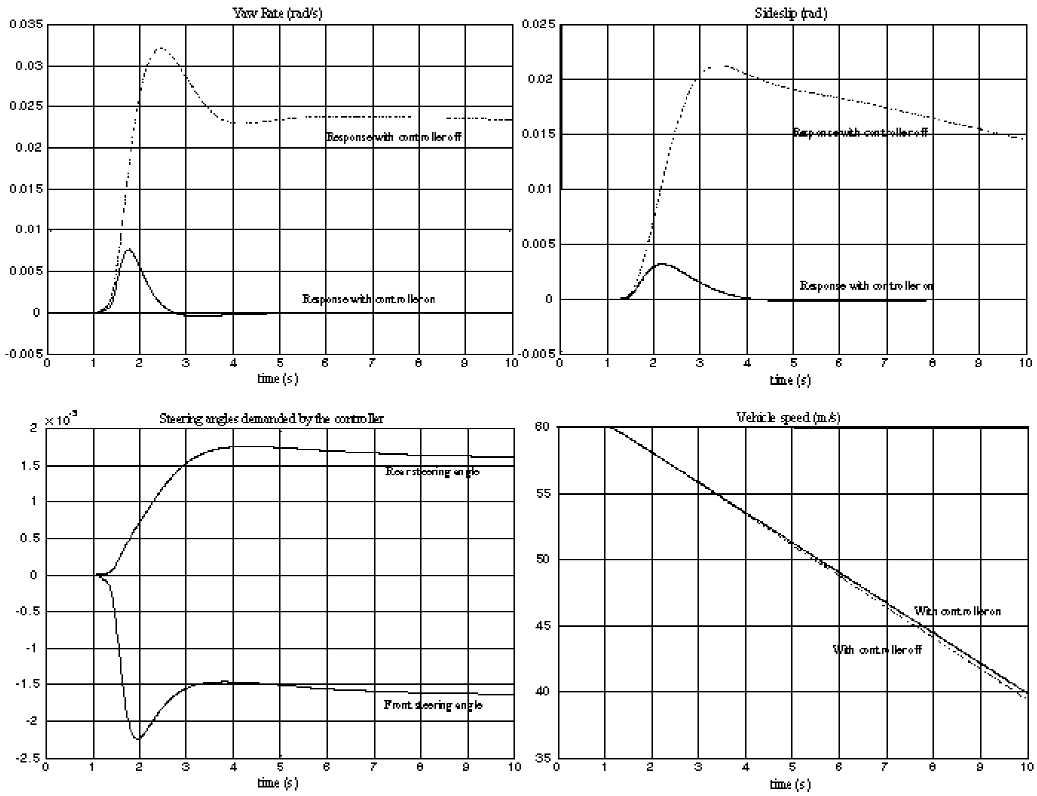


Figure 3: Disturbance rejection performance in simulation of the steering controller in a μ-split braking manoeuvre

the steering controller in this manoeuvre demonstrates the robustness of the control system–the cornering stiffness of a tyre during braking decreases as a result of the longitudinal slip [14]–as well as its ability to operate with varying vehicle speed.

The controller has been implemented on a Mercedes S Class equipped with 4-wheel steer-by-wire. Results from the test drives are shown in Figures 4 to 5. During the test drive results presented in 4, the reference sideslip was set to zero regardless of the driver's inputs, while during the test drive corresponding to Figure 5 the reference signals described the response of a sporty car to the driver's inputs.

Figures 4 and 5 correspond to an implementation of the controller without the feedforward element in the yaw rate reference signal. The experimental results presented above show the robustness of the controller and illustrate its ability to artificially modify the handling behaviour of the vehicle with varying speed.

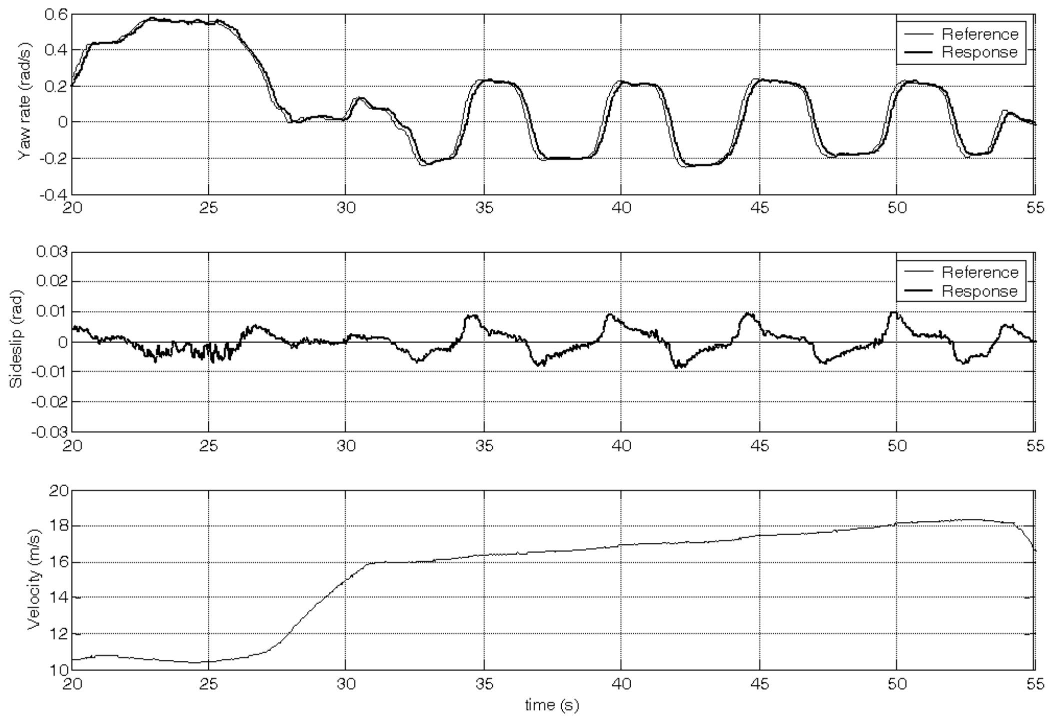


Figure 4: Test results of the test drive with zero sideslip reference.

2.3 Controlling lateral motion through use of other subsystems

We have already mentioned that lateral vehicle motion can be influenced by systems other than the steering system; for example, both braking and suspension systems have been used in the literature by several authors to control these motions. Most of this work has been motivated by vehicle safety considerations. For example, in certain critical situations, when the vehicle tyre limits are reached, the manner in which the vehicle responds to normal driving inputs such as steering angle, changes. In these situations, one may compensate for this effect using the suspension and/or the braking systems. This is basic idea that has been exploited to-date in this area; the main approaches used are individual wheel braking, roll moment distribution and engine torque distribution. It should be noted that active suspension and individual wheel braking have also an effect on lateral dynamics in normal driving conditions; these interactions, while largely ignored in the literature, will be addressed directly during our work on the CEMACS project.

The braking and traction systems

Individual wheel braking and traction control systems are found in many conventional vehicles. Systems of this type are referred to by a variety of names; Electronic Stability Program (ESP) or Vehicle Dynamics Control (VDC). In [15] an in-production VDC from Bosch is described. It uses an Anti-lock Brake System

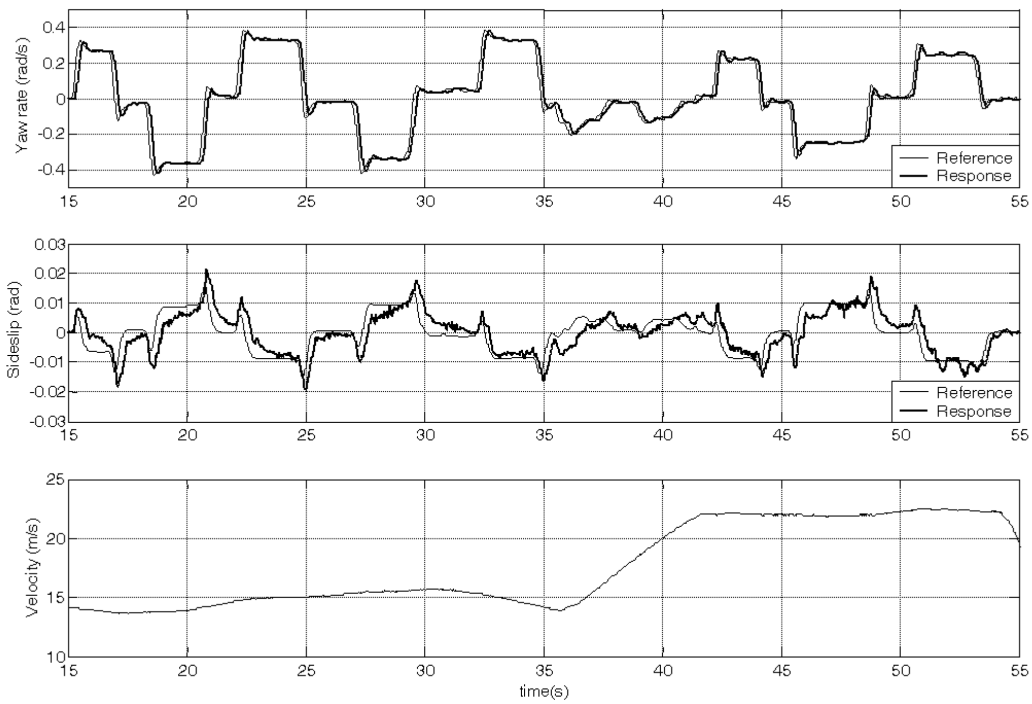


Figure 5: Test results of the test drive with sporty reference.

(ABS) and Traction Control System (TCS) infrastructure to maintain vehicle stability during critical situations and to improve the utilization of friction potential. The controlled forces are the longitudinal tyre forces; these are controlled using individual wheel brake pressures and the engine torque. While this system maintains stability, it has been documented by several authors that the use of individual wheel braking to control yaw-rate is not only uncomfortable for the driver, but also requires to the vehicle to develop very large tyre forces (as illustrated in [16]). Clearly, the use of such large forces to control the vehicle is, in certain situations, undesirable. The paper by Ackerman and his co-authors present simulation results that show that a combination of active front steering together with individual wheel braking on critical situations is more stable and in addition, prevents vehicle roll-over (due to inputs from a normal driver).

The use of individual wheel braking together with active 4-Wheel-Steering is presented in [17]. The stability performance of this system is shown to be be better than the use of the systems operating independently of each other as it makes full use of the vehicle tyre forces.

A system that utilised both torque distribution and active rear steering is presented in [18]. Here the authors exploit the fact that the vehicle cornering force decreases as driving torque increases. The authors demonstrated in a series of publications [19], that the use of torque distribution provides more vehicle stability that active rear steering in slippery surfaces.

The suspension system

It is well known that variations in the vertical load of each wheel influence the total force that can be developed by a tyre. Consequently, the way inertial loads are transferred to the suspension system can be modified using active and semi-active suspension systems with a view to affecting the lateral behaviour of the vehicle.

An analysis of the influence of Roll Moment Distribution(RDC) on improving vehicle stability in near-limit situations is presented in [20]. Simulation results show RDC potential to keep the vehicle out of lateral/directional instability under hard maneuvering conditions.

A semi-active approach that involves the use of varying damping coefficients is presented in [21] (with magneto-rheological dampers). In this paper, the yaw-rate is controlled by through the distribution the damping forces between the front and rear axles thereby improving the vehicle response and reducing driver steering effort. It is also shown that variations in damping force have a big corrective yaw moment potential at high lateral accelerations. RDC is tested experimentally in [22] using an active suspension with a non-linear controller. This controller shows good yaw-rate tracking performance when cornering.

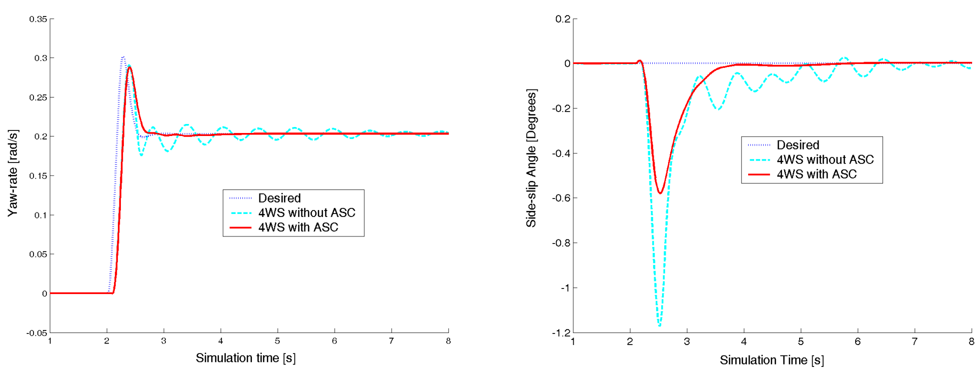
Apart from active suspensions, vehicles equipped with active stabilizing bars at every axle can also influence the suspension forces, distributing their stiffness between the front and the rear axles. In [23] it is shown that the understeer gradient (that defines the lateral behaviour of the vehicle when cornering) can be altered by distributing the roll stiffness between the rear and front than by adding more roll stiffness. By doing so, the lateral force potential of the tyres can be fully exhausted because the wasted potential at the front axle of the vehicle is used. A Proportional-Integral-Derivative (PID) roll moment control together with a Proportional-Integral (PI) roll moment distribution controllers are implemented with active stabilizers in order to reduce any undesired roll and maintain stability.

Vehicle pitch and lift have also an influence on lateral dynamics as it is presented in [24]. Using a frequency characterisation of road disturbances, the authors deduce that the slip angle is correlated to the bounce velocity, while the yaw-rate is related to the pitch velocity, and these relationships appear to be independent of the distribution of suspension spring stiffness between the front and rear axles. The authors develop a control algorithm based on LQR theory for bounce and pitch showing a reduction maximum disturbance influence on side slip and yaw-rate respectively in simulation.

Integrated Braking, traction, and suspension systems

The influence of both, suspension and longitudinal forces, acting as cohesive unit to affect vehicle lateral motion has been also investigated by a number of authors. Recent work in this area is summarised extensively in a paper by Gordon and his co-authors[25]. Roughly speaking, work in this area has proceeded along two lines of enquiry. The first approach is concerned with the design of independently designed systems whose action is coordinated through the use of a supervisory mechanism. For example, the design and analysis of a system constructed in such a manner is analysed in in [26]; here, a control algorithm is implemented to coordinate active control of brakes with magneto-rheological suspension which reduces significantly the brake intervention required to maintain stability in comparison with using only braking. Apart from braking, torque distribution can be controlled to provide stability in critical situations. The effect of engine torque on lateral vehicle motions, and the effect of interaction of between engine torque and the suspension system, are presented in [27].In this paper an active suspension together with variable traction distribution is used to improve handling and stability. Even if RDC with the active suspension has a better yaw-rate tracking performance, the integrated controller is shown to have better stability and handling than the those with the respective subsystems acting independently. All systems, Active suspension, 4-wheel-steering and VDC, are tested independently and coordinated in [28] using a H_∞ control algorithm. The integrated control provides (in simulation) significantly different performance and more stability than the uncoordinated controllers showing that importance of interactions. Better performance of an integrated approach over single approach is demonstrated using test vehicle and results presented in [29]. Here ESP is integrated with other controllers like active front steering (AFS), and the resulting system is shown a significant improvement in lateral dynamic handling and is shown to require less drivers intervention and less braking pressure required to maintain stability. In addition, the results also show that these interactions can also used to affect longitudinal dynamics; for example, to decrease braking distances.

Finally, using the 4-wheel steering controller developed at the Hamilton Institute by M.Vilaplana [9], the influence of vertical dynamics on lateral dynamics was tested by simulating a step in the steering wheel with and without an active suspension controller (ASC) activated. The results presented in Figure 6 depict the influence of roll vertical dynamics on the steering system tracking performance. The roll dynamics affect mainly the side-slip angle and induce an oscillation in the yaw-rate.



(a) (b)

Figure 6: Vertical dynamics influence on (a) Yaw-rate and (b)Side-slip response of 4-Wheel Steering Controller

3 Control of Vertical dynamics

Traditionally, the vertical dynamics of an automobile are controlled using the suspension system. Suspension systems are usually designed with three objectives in mind; (i) to isolate the vehicle cabin from road disturbances; and (ii) to insulate the vehicle body from load disturbances (inertial loads induced by braking and cornering) and influence the cornering properties of the vehicle. These requirements, referred to frequently as ride and handling, become conflicting requirements when only conventional passive suspension systems are deployed (such as spring and shock absorbers), but can satisfied independently of each other when active systems are used. Clearly, a well designed suspension system should adequately address each of these design considerations. In this section we provide a very brief overview of research on this topic. As we are interested in affecting both ride and handling, we consider here only work that has been carried out in the area of active suspensions. As well as a large body of literature, several excellent surveys have written on this topic. We refer the interested reader to the work by Hrovat[30], and the book by Gillespie[14], as an entry point to this extensive topic.

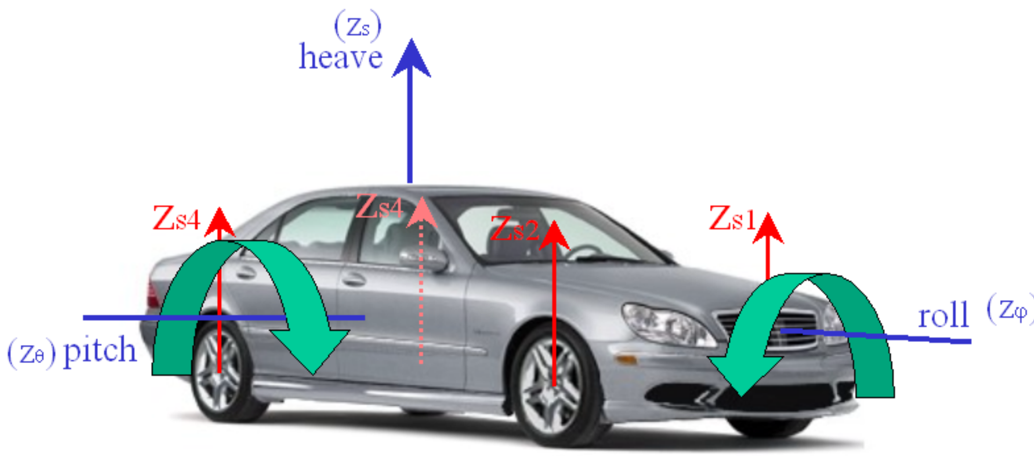


Figure 7: Vehicle Vertical Dynamics

Vehicle Ride is normally associated the perception of vehicle disturbances as experienced by the vehicle passengers. There are several components that contribute to vehicle ride; the most important of which are road disturbances that are transmitted to the vehicle via the suspension system. From the point of view of suspension design, vehicle ride is the perception of the frequency spectrum of vehicle vibrations below 25 Hz that are produced by road roughness and on-board sources, being the former the most relevant. On the other hand, Vehicle Handling refers to the ability of the vehicle/driver combination to manoeuvre [14]. A vehicle's handling characteristics are greatly influenced by forces and moments generated by inertial and aerodynamic loadings such as those caused by braking, cornering and wind gusts.

Suspension system hardware can range from completely passive to completely active depending on the degree of force control available to the driver, as illustrated in Fig. 8. The mechanical model depicted corresponds to a quarter-car model. It is composed of: a quarter of the vehicle mass called the sprunged mass m_s, a wheel mass called unsprunged mass m_u, the suspension system between m_u and m_s, and the tyre represented as a spring between m_u and the uneven road. For a detailed explanation of the quarter-car model, we refer the interested reader to [31].

The semi-active suspension illustrated in Fig. 8 is a damper with continuously variable (or regulated) damping coefficient instead of a passive one. As the damping force is proportional to the vertical velocity of the suspension displacement, semi-active suspensions can only affect transient behavior of the suspension deflection and not its steady-state. Besides semi-active damper, there are also semi-active spring systems systems where the spring stiffness changes by means of pneumatic or hydraulic pressure [32]. Active systems, on the other hand, influence both steady-state as well as transient behavior of suspension deflection. These systems are usually classified according to the maximum response frequency or bandwidth of the actuators that they employ. Low bandwidth active systems are able control the force at the suspension spring, and indirectly in combination with with a shock absorber, the suspension force. Typically, such systems works using hydraulic power and operate at frequencies below 5 Hz.

Medium bandwidth systems also typically employ hydraulic power and can control suspension deflection directly at up to a maximum frequency of 30 Hz. Systems of the form described above have been already implemented in production units. Finally, high bandwidth systems with a response of up to 100 Hz may be achieved with an electromagnetic actuator. [33].

The vertical movement of the vehicle (heave) together with its inclinations (pitch and roll) illustrated in Fig. 7 constitute the vehicle vertical dynamics. These are mainly defined by the suspension system used in the vehicle. The vehicle manufacturer may choose between a passive, semi-active or active suspension. The choice of a particular suspension system depends usually on issues, including power consumption, comfort and performance requirements. As Active suspensions consume a large amount of energy ing [32], they are used mainly in high-end vehicles. Semi-active suspensions are the system of choice when low energy consumption is a priority; in many cases semi-active systems yield performance that is close to the active one in many situations.

The design of suspension control systems has been an active topic of research for more than four decades, and since the end of the 1980's active suspensions have been a feature of production automobiles. The design of controllers for active and semi-active suspensions has been approached using many different methods. These range from classical techniques to modern methods such as Youla parameterization [34]. As the ride optimisation problem can be viewed as an optimal filtering problem (where one would attempt to eliminate the negative effects of vibrations caused by road roughness) many techniques from optimal control theory have also been used in this

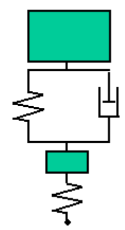
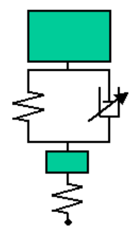
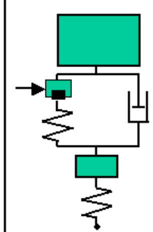
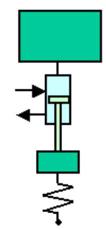
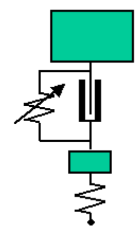
System	passive	semiactive	slow active	full active	full active
Mechanical Model					
Components	Spring with hydraulic damper and stabilisator	Continuously (regulated) variable dampers coefficient < 50 Hz	Hydraulic energy input in parallel with damper 0 – 5 Hz	Direct hydraulic energy input 0 – 30 Hz	Electric power controlling electromagn. force input 0 – 100 Hz

Figure 8: Classification of Suspension Systems[33]

area. As we have already mentioed, an extensive survey of vehicle active suspension control, with particular focus on optimal control theory, is presented in [30]. One of the control concepts frequently found during our brief literature survey was Skyhook and its variations. Skyhook control has the objective of damping the vehicle body (m_s) motion from road and other perturbations, as if a damper was placed between m_s and a fixed point[30] as depicted in Fig. 9. Similarly, Groundhook control differs from Skyhook as the damper is placed between the wheel (m_u) and the fixed point. There is also the hybrid version using both: skyhook and groundhook concepts, among others.[35].

Semi-active suspensions are an attractive alternative to active systems in many situations with the advantage of requiring smaller use of power [32]. Several techniques for the design of such systems are reviewed in [35]. Similarly, the limits and performance possibilities of semi-active suspensions with variable damping and stiffness coefficients are studied in [32]. Semi-active spring systems were difficult to find in the literature survey, but they are already in production vehicles (the Electronic Air Suspension from Land Rover's Range Rover). Systems with fixed damping coefficients are analysed and modelled as switched linear systems in [36]. New actuators called CVTs for Continuous Variable Dampers have recently become popular in semi-active suspension design. Such systems are particularly suited to the used of continuous time control design methods. In [37] a fuzzy control systems is presented. The focus of this work is to reduce the effect of road perturbations and and to simultaneously realise low chassis movement and tire movement. A mixed H_2/H_∞ controller for a CVT-based system are presented in [38]. In this paper, using a full vehicle model, a H_2 controller is used to achieve a system to control

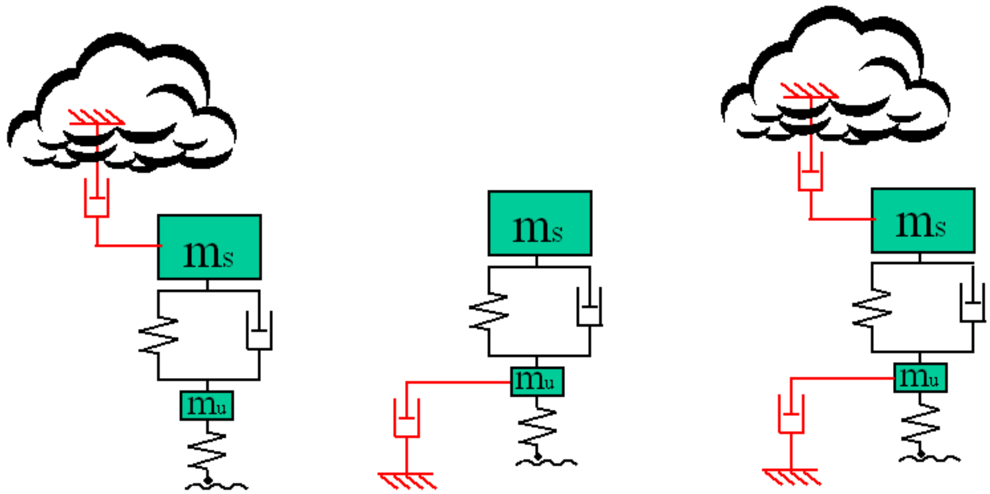


Figure 9: (a) Skyhook, (b) Groundhook and (c) Hybrid Skyhook/Groundhook control objectives

vertical chassis accelerations, where as a H_∞ controller is used to control vertical deflection velocity. The low cost and low power requirements of semiactive systems has made their inclusion in production units possible; not only high end vehicles like Cadillac's 2002 Seville STS or Maserati's 2002 Spyder but also in small size vehicles like Opel's 2004 Astra.

Control design techniques for active and semi-active systems may be categorised in several ways. A particularly convenient way to examine the work that has gone on in the area is to use the type of vehicle model upon which the control design is based; namely, whether a quarter car, a half car, of a full car model has been used to design the control system.

Again we note that much of of the work on active suspension design is presented in [30]. Historically, initial attempts to design active suspension systems involved using two degree of freedom (2DoF) quarter car models. Such designs are not only of historical interest; they are also still used today. For example, many recent studies [39][40] have been based on the classical 2DoF model. Using this model it is possible to illustrate intuitiely the skyhook and groundhook concepts used by many authors: see [41],[42],[43],[35] and [30] for more details about skyhook and groundhook. In [41], the 2DoF model is used as a basis of the sliding model control system that is shown experimentally to have better ride performance near the chassis natural frequency. To take into account for changes in conditions of the road and avoid to reach the actuator limits, a controller based on nonlinear backstepping is used by [44] and [45]. This controller is also based on a relatively simple 2DoF model. In such a paper, the vehicle suspension varies from soft (to decrease lateral acceleration) and hard (to decrease actuators displacement). Such a vehicle would be comfortable on a wide variety of road surfaces, and become hard on particularly bumpy surfaces. A similar approach is used in [40]; here the authors again use nonlinear backstepping to achieve a system that shown better response to bumps in the road than a conventional passive suspension. Ting et al. [46] use a Fuzzy controller based on sliding mode control theory to minimize spring mass acceleration, suspension deflection and tyre fluctuation, to improve vehicle handling. Finally, [34] uses the 2DoF model to design a linear controller that results in improved vehicle handling (by reducing effects of inertial perturbations such as the lateral acceleration when cornering). In this paper, road and inertial disturbance paths are separated so responses can be adjusted independently.

Most of the recent work in the area has been based on controllers that are designed using more complex vehicle models. In particular, a half vehicle model with 4 degrees of freedom (4DoF) is used by a number of authors; see for example [34] and [47]. Such models are used in applications where it is desired to control vehicle pitch and heave. Full vehicle models with 7 degrees of freedom (7DoF) capture more vehicle behaviours and used when not only heave and pitch are to be controlled, but also vehicle roll and warp [34]. A full vehicle model is used in [42]. Here the authors demonstrate improved ride performance using a feedback controller. Similarly, in [39], vehicle ride is improved (in simulation results) when compared to a passive system using a controller based on mixed μ synthesis. Handling as well as ride are improved in car tests in [43] and [47] using an H^{∞} control that is designed to be robust to parameter uncertainty. The vehicle response to road perturbations is improved near the chassis natural frequency, and better handling performance is achieved by a smaller roll response to lateral accelerations. The controller presented in [47] has the advantage of being simpler and easier to implement as the model used is decoupled into two 4DoF models. In M.Smith's paper [34], control effort is focused on vehicle handling. A 7DoF model is also used in this paper, but this model is later decoupled into two 4DoF models and subsequently, each of these, under certain (unrealistic) assumptions, into four 2DoF models. Nevertheless, the controller parametrizations designed for both, 2DoF and 4DoF models, and tested on a nonlinear dynamical model simulation, show very good rejection of inertial perturbations.

Even though active suspensions require actuators for their physical implementation, many papers neglect them and not include them in the model to be controlled. The actuators that were found in the literature where hydropneumatic and electrohydraulic [30]. In most cases, electrohydraulic nonlinear models are linearized like in [39]. Only in [43], an linearized hydropneumatic actuator was used.

Apart from actuators, other aspects that affect vertical and lateral dynamics include elastokinematics and the suspension geometry. In almost every case paper that we surveyed such effects were neglected. The term 'Elastokinematics' refers to the influence of rubber couplings that are used to damp vibrations coming from the tyres. These affect the steering and suspension subsystems. The interested reader is referred to [48] for more about this subject. Simulation results illustrating the influence of elastokinematics on lateral dynamics are presented in [49]. The suspension geometry on the other hand appears to have been almost completely neglected in the literature.

Finally, except for [50], we found no paper that focusses on the tracking problem; rather the majority of papers appear to focus on the regulation problem.

4 Integrated Dynamics Model

Vehicle dynamics define vehicle behaviour and are concerned with the accelerated movement of a vehicle subject to the forces imposed by the tyres, gravity and aerodynamics. Consider, for example, the tyre forces developed to keep the vehicle on track when the vehicle is cornering at a determined radii. As long as the centripetal forces at the tyres are proportional to the vehicle centrifugal acceleration, the vehicle will keep its track.

This section describes the modelling of the lateral and vertical dynamics of a vehicle equipped with 4-wheel steering and active suspension. The modelling is simplified into two submodels: lateral and vertical, which are later integrated into a single model.

4.1 Lateral Dynamics Submodel

Consider a vehicle from above with the center of gravity (CoG) positioned with respect to the contact point of every tyre[1] and is illustrated in Figure 10. The following assumptions are made: the velocity, side-slip and yaw-rate are at the CoG, all the tyres have an independent steering angle, the forces developed at the tyres act at their contact point, and the longitudinal velocity v_x is constant. As only lateral dynamics are going to be modelled, the longitudinal tyre forces are neglected; i.e. no breaking or tractive moments at the tyres.

The lateral dynamics can be described by the 3-tuple (v_x,v_y,$\dot{\Psi}$)[51] and as the longitudinal velocity v_x is considered to be constant, the lateral dynamics will be defined by the lateral velocity v_y and the yaw-rate $\dot{\Psi}$.

Let us start by considering the vehicle from the inertial coordinate system, i.e. a pure translation from the fixed coordinate to the Center of Gravity of the vehicle. Newton's second law at the inertial coordinate system (x_0, y_0), as shown in Figure 11, is obtained as follows:

$$m\frac{d}{dt}\begin{pmatrix} v_{x0} \\ v_{y0} \end{pmatrix} = f_0 \tag{1}$$

where m is the mass of the vehicle (including the mass of the tyres), v_{x0} and v_{y0} are the components of v in the inertial coordinate system and f_0 are the forces acting at the CoG in the inertial coordinate system. To transform from the vehicle coordinate system (x, y) to the inertial coordinate system (x_0, y_0) , the matrix $D(\Psi)$ is used on the forces f:

[1]Tyres have contact areas or patches were the forces are distributed. The total forces are assumed to be at the tyre contact point.

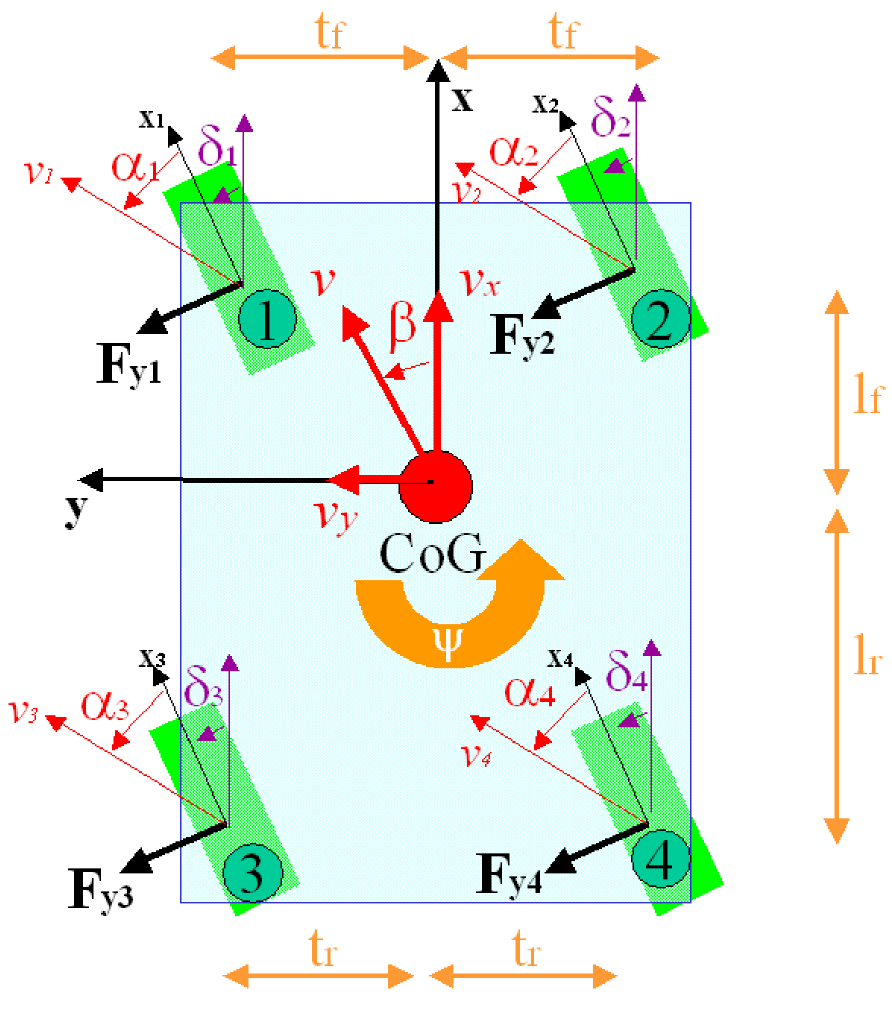


Figure 10: Lateral dynamics model with a tyre index for every wheel ($i = 1, 2, 3, 4$)

$$f_0 = D(\Psi) f \tag{2}$$

and velocities:

$$\begin{pmatrix} v_{x0} \\ v_{y0} \end{pmatrix} = D(\Psi) \begin{pmatrix} v_x \\ v_y \end{pmatrix} \tag{3}$$

where:

$$f_0 = \begin{pmatrix} f_{x0} \\ f_{y0} \end{pmatrix},$$

$$f = \begin{pmatrix} f_x \\ f_y \end{pmatrix}, \text{ and}$$

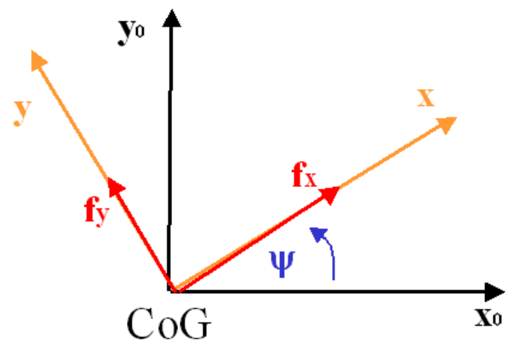


Figure 11: Inertial Coordinate System

$$D(\Psi) = \begin{pmatrix} cos\Psi & -sin\Psi \\ sin\Psi & cos\Psi \end{pmatrix}.$$

Substituting (2) and (3) in (1):

$$m\frac{d}{dt}\left(D(\Psi)\begin{pmatrix} v_x \\ v_y \end{pmatrix}\right) = D(\Psi)f$$

and derivating the product inside the parenthesis:

$$m\left[D(\Psi)\begin{pmatrix} \dot{v}_x \\ \dot{v}_y \end{pmatrix} + \frac{d}{dt}(D(\Psi))\begin{pmatrix} v_x \\ v_y \end{pmatrix}\right] = D(\Psi)f$$

the matrix $D(\Psi)$ can be taken out of the parenthesis:

$$mD(\Psi)\left[\begin{pmatrix} \dot{v}_x \\ \dot{v}_y \end{pmatrix} + \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\dot{\Psi}\begin{pmatrix} v_x \\ v_y \end{pmatrix}\right] = D(\Psi)f$$

and we can see from this that it is easy to go back to the vehicle coordinate system by multiplying both sides of the equation by the inverse of the rotational matrix $D^{-1}(\Psi)$:

$$m\left[\begin{pmatrix} \dot{v}_x - \dot{\Psi}v_y \\ \dot{v}_y + \dot{\Psi}v_x \end{pmatrix}\right] = f. \tag{4}$$

Considering f to be the addition of all the vector forces at the tyres f_i where:

$$f_i = \begin{pmatrix} f_{xi} \\ f_{yi} \end{pmatrix} = \begin{pmatrix} -sin\delta_i F_{yi} \\ cos\delta_i F_{yi} \end{pmatrix}$$

and i stands for the tyre index (1,2,3,4), the equation (4) can be rewritten considering only the lateral dynamics as:

$$m\left(\dot{v}_y + \dot{\Psi}v_x\right) = \sum_{i=1}^{4}\left(cos\delta_i F_{yi}\right). \tag{5}$$

i	r_x	r_y
1	l_f	t_f
2	l_f	$-t_f$
3	$-l_r$	t_r
4	$-l_r$	$-t_r$

Table 1:

From Eq. (5) we may obtain the first equation describing the lateral dynamics:

$$\dot{v}_y = -\dot{\Psi}v_x + \frac{1}{m}\sum_{i=1}^{4}(cos\delta_i F_{yi}) \tag{6}$$

It is also possible to write Equation (6) in terms of the vehicle side slip. The sideslip angle at the CoG (β) is defined as:

$$tan\beta = \frac{v_y}{v_x} \tag{7}$$

where v_x and v_y are the projection of the v vector to the x and y axis respectively. If we derive Equation (7) with respect to time (recall that v_x is constant):

$$\dot{\beta}\frac{1}{cos^2\beta} = \frac{\dot{v}_y}{v_x} \tag{8}$$

the Equation (6) may be rewritten as:

$$\dot{\beta} = -\dot{\Psi}cos^2\beta + \frac{1}{mv_x}\sum_{i=1}^{4}(cos\delta_i F_{yi})\, cos^2\beta \tag{9}$$

So let us obtain now the equation for the yaw-rate. The sum of moments at the center of gravity generated by the forces at the tyres is:

$$I_{zz}\ddot{\Psi} = \sum_{i=1}^{4}\left[\begin{pmatrix} -r_{yi} & r_{xi} \end{pmatrix}\begin{pmatrix} f_{xi} \\ f_{yi} \end{pmatrix}\right] = \sum_{i=1}^{4}(r_{yi}sin\delta_i F_{yi} + r_{xi}cos\delta_i F_{yi}) \tag{10}$$

where I_{zz} is the moment of inertia about the z axis; and r_{xi} and r_{yi} are the x and y components of the tyre position. The values of r_{xi} and r_{yi} are given in the following table:
where the dimensions l_f and l_r are depicted in Figure 10.

From Eq. (10) it is easy to obtain the second equation for the lateral dynamics:

$$\ddot{\Psi} = \frac{1}{I_{zz}}\sum_{i=1}^{4}(r_{yi}sin\delta_i F_{yi} + r_{xi}cos\delta_i F_{yi}) \tag{11}$$

At this point, the lateral dynamics are described by either pair of Equations (6) and (11) or Equations (9) and (11). Nevertheless, the lateral forces F_{yi} developed at every tyre are unknown so equations in function of the steering angle inputs and the lateral dynamic variables are necessary.

Let us start by obtaining the velocity at every tyre (v_i):

$$v_i = v + \dot{\Psi} \times r_i$$

where v_i, $\dot{\Psi}$ and r_i are vectors, i stands for the tyre index (1,2,3,4), $\times$ represents a cross product and r_i is the position vector from the acronym:CoG to the tyre contact point. Then, the angle (β_i) between the velocity vector v_i and the x-axis is obtained as follows:

$$tan\beta_i = \frac{v_y + \dot{\Psi} r_{xi}}{v_x - \dot{\Psi} r_{yi}} \tag{12}$$

where r_{xi} and r_{yi} are the x and y components of the tyre position defined in Table 1.

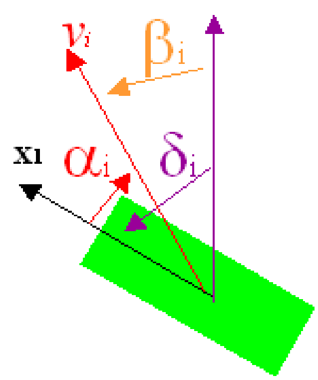


Figure 12: Tyre angles

Now we may define the side-slip angle of every tyre accordingly to the Figure 12 as:

$$\alpha_i = \delta_i - \beta_i \tag{13}$$

where δ_i is the input steering angle of the tyre i for $i = 1, 2, 3, 4$. Finally, in order to consider the load sensitivity effect [14]p.215, the side-force is modelled as a nonlinear function of the side-slip angle at every tyre and the normal force F_{zi}:

$$F_{yi} = \alpha_i \left(C_{i1} F_{zi} - C_{i2} F_{zi}^2 \right) \tag{14}$$

where C_{i1} and C_{i2} are positive constants. Substituting Equations (12) and (13) in (14) the vertical forces at the tyres may be rewritten as:

$$F_{yi} = \left[\delta_i - arctan\left(\frac{vsin\beta + \dot{\Psi} r_{xi}}{vcos\beta - \dot{\Psi} r_{yi}} \right) \right] \left(C_{i1} F_{zi} - C_{i2} F_{zi}^2 \right) \tag{15}$$

Bicycle Model

The bicycle model or one-track model is presented as it is used in most of the literature we found on steering and it was also used by M.Vilaplana [52] in the 4-wheel-steering controller developed at the Hamilton Institute. It is a simplification of equations (9) and (11) with $r_{xi} = 0$ for $i = 1, 2, 3, 4$ (i.e. the width of the vehicle is zero)and linearized for small steering and side-slip angles:

$$\dot{\beta} = -\dot{\Psi} + \frac{(S_f + S_r)}{mv_x}, \text{ and} \tag{16}$$

$$I_{zz}\ddot{\Psi} = S_f l_f - S_r l_r, \tag{17}$$

where S_f and S_r are the side forces at the front and rear axle, respectively, and defined as:

$$S_f = F_{y1} + F_{y2}, \text{ and}$$

$$S_r = F_{y3} + F_{y4}.$$

4.2 Vertical Dynamics Submodel

Consider a vertical dynamics model with 7 degrees of freedom as depicted in Fig. 13 and described in [53]. It includes the vehicle body as a stiff body and the four wheels as point masses. For the vehicle body has 3 degrees of freedom at the center of gravity: heave z_s or vertical motion, pitch angle (z_θ) or rotational motion about the y-axis, and roll angle(z_ϕ) or rotational motion about the x-axis. Each wheel position z_{ui} changes only in the vertical direction for i representing the wheel index $i = 1, 2, 3$ and 4. Let us consider that the mass of the chassis (m_s), sometimes referred to as sprung mass, does not include those of the tyres (m_i) also called unsprung masses.

The suspension structure is considered weightless with the geometry depicted in Fig. 13. It has two contact points or joints with free rotation at the vehicle body and another at the wheel. The suspension strut at its middle is considered vertical, and a linear damper is mounted in parallel to a spring and an actuator. The actuator acts directly on the spring and the latter is considered to have a linear behaviour.

The wheels are considered to move only in vertical direction and the tyres are modelled as vertical linear springs between the wheel point mass at z_{ui} and the road perturbations z_{ri}, being the damping influence not considered. Besides, their relative longitudinal and lateral motions with respect to the body are not considered. Moreover, the actuator dynamics are not considered and the input taken as the displacement of its piston (u_i), being positive for expansion from the initial condition.

The outside forces acting on the vehicle at the CoG are considered to be the acceleration of gravity (g) for the vertical motion, and lateral and longitudinal accelerations for the roll and pitch angular motions, respectively. Nevertheless, as the vehicle is considered to be always on a flat surface, the gravity will not be considered explicitly but implicitly in the initial displacement of the linear springs. Besides, even if the vehicle is assumed to roll and pitch at the center of gravity, the influence of the lateral accelerations consider the roll and pitch axes to be below the CoG by the distance h_r and h_p respectively. It should be noted that a negative longitudinal acceleration a_x, i.e. vehicle braking, will produce a positive pitch moment while a negative lateral acceleration a_y, a negative roll moment because of the body inertial behaviour to the forces produced at the tyres.

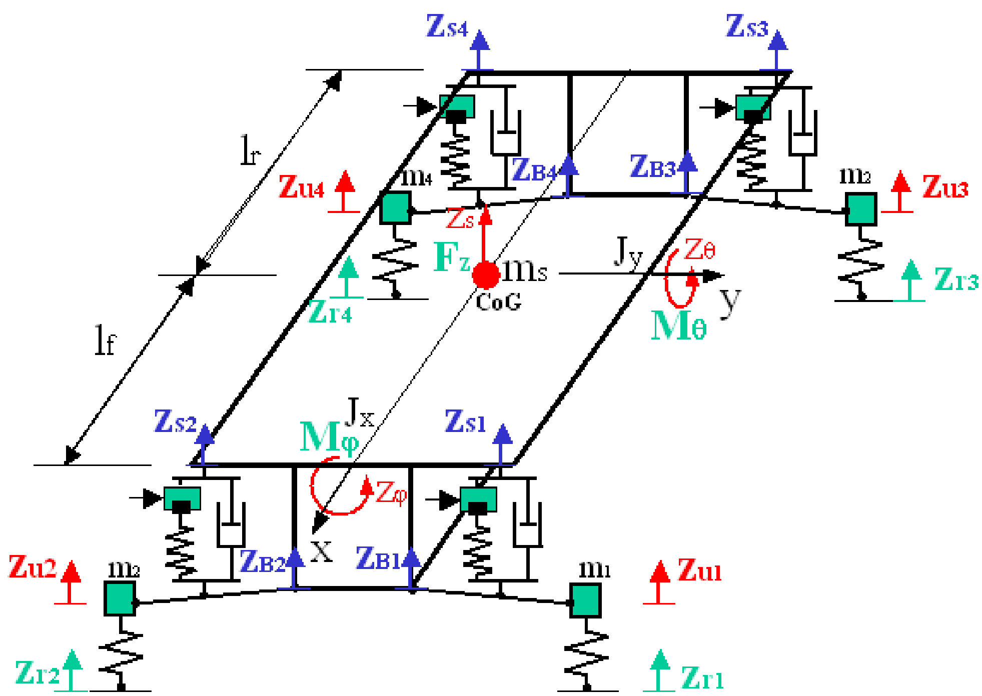


Figure 13: Vertical Dynamics Model

Vehicle chassis vertical dynamics

The equation of motion of the vehicle chassis as depicted in Figs. 13 and 14 are obtained using Newton's second law for the vertical motion and the equations of torque balance (about x- and y-axes):

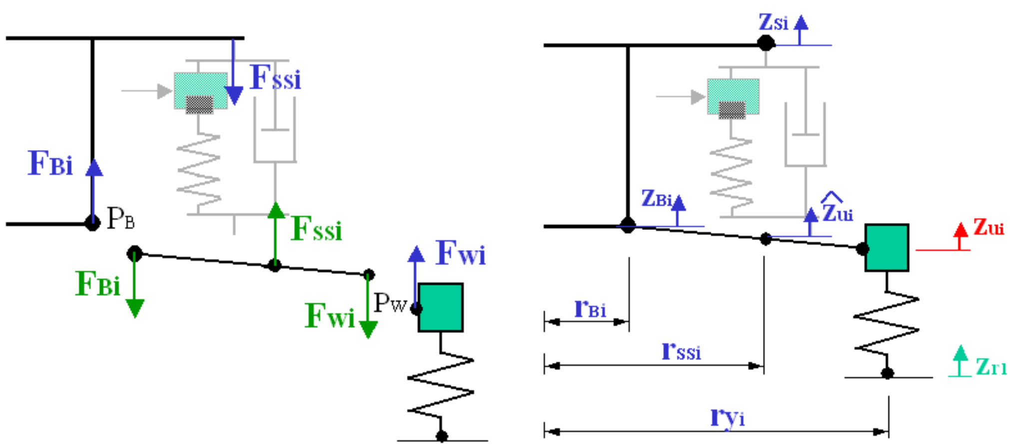
$$m_s \ddot{z}_s = \sum_{i=1}^{4} (F_{Bi} - F_{ssi}) + F_z \tag{18}$$

$$I_\theta \ddot{z}_\theta = \sum_{i=1}^{4} (r_{xi}(F_{ssi} - F_{Bi})) + M_\theta \tag{19}$$

$$I_\phi \ddot{z}_\phi = \sum_{i=1}^{4} (r_{Bi}F_{Bi} - r_{ssi}F_{ssi}) + M_\phi \tag{20}$$

i	r_{Bi}	r_{ssi}
1	l_{bf}	l_{ssf}
2	$-l_{bf}$	$-l_{ssf}$
3	l_{br}	l_{ssr}
4	$-l_{br}$	$-l_{ssr}$

Table 2:



(a) (b)

Figure 14:

where k_i is the stiffness coefficient of the spring of the suspension number i, c_i is the stiffness coefficient of the damper at the suspension number i, F_{ssi} are the forces from the suspension strut acting on the vehicle body, F_{Bi} are the forces at the body bearing mount, F_z is the vertical perturbation at the CoG, M_θ is the moment perturbation for pitch, M_ϕ is the moment perturbation for roll, the dimensions r_{xi} were given in Table 1 while the values for r_{Bi} and r_{ssi} are in Table 2.

I_θ and I_ϕ are the moments of inertia about the roll and pitch axes and, according to Steiner's theorem:

$$I_\theta = J_y + m_s h_p^2 \tag{21}$$

$$I_\phi = J_x + m_s h_r^2 \tag{22}$$

where J_x and J_y are the moments of inertia at the CoG about the x- and y-axes, respectively.

Let us consider now the geometry of the suspension in order to obtain the relationships between the forces at the bearing mount F_{Bi} and those at the suspension struts F_{ssi}. By using the equilibrium of moments at wheel point P_w (see Fig. 14):

$$F_{ssi}(r_{yi} - r_{ssi}) - F_{Bi}(r_{yi} - r_{Bi}) = 0, \tag{23}$$

and by solving for F_{Bi}:

$$F_{Bi} = \frac{r_{yi} - r_{ssi}}{r_{yi} - r_{Bi}} F_{ssi}, \tag{24}$$

the relationship may be written as:

$$F_{Bi} = (1 - r_{Di}) F_{ssi}, \tag{25}$$

with:

$$r_{Di} = \frac{r_{ssi} - r_{Bi}}{r_{yi} - r_{Bi}}. \tag{26}$$

Substituting the Equation (25) in Equations (18),(19) and (20), and considering the outside forces, the vertical dynamics of the body may be rewritten as:

$$m_s \ddot{z}_s = -\sum_{i=1}^{4} (r_{Di} F_{ssi}) \tag{27}$$

$$I_\theta \ddot{z}_\theta = \sum_{i=1}^{4} (r_{xi} r_{Di} F_{ssi}) - m_s h_p a_x \tag{28}$$

$$I_\phi \ddot{z}_\phi = \sum_{i=1}^{4} \left(\left[(1 - r_{Di}) r_{Bi} - r_{ssi} \right] F_{ssi} \right) + m_s h_r a_y \tag{29}$$

Wheel vertical dynamics

In order to obtain the tyre dynamics, we require to consider once again the geometry of the suspension to obtain the forces at the wheels, as well as its displacements. By obtaining the equilibrium of moments at bearing mouting point P_B (see Fig. 14):

$$F_{ssi}(r_{ssi} - r_{Bi}) - F_{Wi}(r_{yi} - r_{Bi}) = 0, \tag{30}$$

and by solving for F_{Wi} we obtain the vertical force at the tyre F_{Wi} as a function of the force at the suspension strut F_{ssi}:

$$F_{Wi} = r_{Di} F_{ssi}. \tag{31}$$

Now let us consider the displacement at the lower part of the suspension strut $\hat{z}_{ui}$ as depicted in the Figure 14. As only vertical displacements are considered and now relative displacement between the body and the wheels are considered, by similar triangles:

$$\frac{z_{ui} - z_{Bi}}{r_{yi} - r_{Bi}} = \frac{\hat{z}_{ui} - z_{Bi}}{r_{ssi} - r_{Bi}}, \tag{32}$$

and solving for $\hat{z}_{ui}$:

$$\hat{z}_{ui} = z_{Bi} + \frac{r_{ssi} - r_{Bi}}{r_{yi} - r_{Bi}} (z_{ui} - z_{Bi}), \tag{33}$$

which may be rewritten as:

$$\hat{z}_{ui} = z_{Bi} + r_{Di}(z_{ui} - z_{Bi}). \tag{34}$$

Recalling that the vehicle chassis is a stiff body, by kinematics the vertical velocities at the chassis contact points with the suspension $\dot{z}_{si}$ and $\dot{z}_{Bi}$ are obtained by:

$$\dot{z}_{Bi} = \dot{z}_s + r_{xi}\dot{z}_\theta - r_{Bi}\dot{z}_\phi, and \tag{35}$$

$$\dot{z}_{si} = \dot{z}_s + r_{xi}\dot{z}_\theta - r_{ssi}\dot{z}_\phi; \tag{36}$$

On the other hand, the contact point displacements z_{Bi} and z_{si} are obtained by considering small pitch and roll angles and linearizing:

$$z_{Bi} = z_s + r_{xi}z_\theta - r_{Bi}z_\phi, and \tag{37}$$

$$z_{si} = z_s + r_{xi}z_\theta - r_{ssi}z_\phi; \tag{38}$$

At this point, all positions and velocities at the suspension strut are available and the force at the suspension struts F_{ssi} is:

$$F_{ssi} = c_i(\dot{z}_{si} - \dot{\hat{z}}_{ui}) + k_i(z_{si} - u_i - \hat{z}_{ui}) \tag{39}$$

where u_i is the displacement of the actuator.

Finally, recalling the vertical forces at the wheels as in Equation (31), the wheel motion z_{ui} is obtained using Newton's second law:

$$m_i\ddot{z}_{u_i} = r_{Di}F_{ssi} + k_{ti}(z_{ri} - z_{ui}) \tag{40}$$

where k_{ti} is the stiffness coefficient of the spring of the tyre index i and z_{ri} are the road perturbations at contact point of tyre i.

4.3 Integrated Model

The interactions of the lateral dynamics into the vertical ones are given by the lateral acceleration a_y defined as:

$$a_y = \dot{v}_y + \dot{\Psi}v_x, \tag{41}$$

which is dependent of the complete lateral dynamics (Equations (6) and (11)) and affects the vertical dynamics for roll (Equation (29)). On the other side, the interactions from the vertical dynamics on the lateral ones are through the forces a the tyres F_{zi} assumed for flat surfaces as:

$$F_{zi} = k_{ti}z_{ui}. \tag{42}$$

It may be noticed from 42 that each of the vertical tyre displacements z_{ui} (Equation (40)) influence the lateral forces developed at one tyre F_y (Equation (15)) and thus the whole lateral dynamics.

The integrated model is rewritten here for the lateral dynamics:

$$\dot{v}_y = -\dot{\Psi}v_x + \frac{1}{m}\sum_{i=1}^{4}(cos\delta_i F_{yi}) \tag{43}$$

$$\ddot{\Psi} = \frac{1}{I_{zz}}\sum_{i=1}^{4}(r_{yi}sin\delta_i F_{yi} + r_{xi}cos\delta_i F_{yi}) \tag{44}$$

$$F_{yi} = \left[\delta_i - arctan\left(\frac{vsin\beta + \dot{\Psi}r_{xi}}{vcos\beta - \dot{\Psi}r_{yi}}\right)\right]\left(C_{i1}k_{ti}z_{ui} - C_{i2}(k_{ti}z_{ui})^2\right) \tag{45}$$

and for the vertical dynamics

$$m_s\ddot{z}_s = -\sum_{i=1}^{4}(r_{Di}F_{ssi}) \tag{46}$$

$$I_\theta\ddot{z}_\theta = \sum_{i=1}^{4}(r_{xi}r_{Di}F_{ssi}) - m_s h_p a_x \tag{47}$$

$$I_\phi\ddot{z}_\phi = \sum_{i=1}^{4}([(1-r_{Di})r_{Bi} - r_{ssi}F_{ssi}]) + m_s h_r\left(\dot{v}_y + \dot{\Psi}v_x\right) \tag{48}$$

$$m_i\ddot{z}_{u_i} = r_{Di}F_{ssi} + k_{ti}(z_{ri} - z_{ui}) \tag{49}$$

with F_{ssi} as in Equation (39).

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